

HOW STRONG IS A COUNTERFACTUAL?*

There are two leading theories about the meaning of counterfactuals like (1):

- (1) If David's alarm had not gone off this morning, he would have missed class.

Both say that (1) means, roughly, that David misses class in all of the closest worlds where his alarm does not go off. Both theories agree that this set of worlds—the *counterfactual domain*—depends partly on the world of evaluation and partly on the context in which (1) is uttered. Still, they disagree about how exactly the counterfactual domain is calculated. The *Variably Strict Account* (VSA) says that the counterfactual domain can also depend on the antecedent of the counterfactual; it can *vary* from antecedent to antecedent, even when we hold the world of evaluation and context of utterance fixed. The *Strict Account* (SA) says that the counterfactual domain does not depend on the antecedent of the counterfactual; once we have fixed the world of evaluation and context of utterance, we have also fixed the counterfactual domain.

VSA and SA validate different inference patterns. Perhaps most famously, they disagree about a principle known as *Antecedent Strengthening*. SA validates the principle; VSA does not. Early VSA theorists, such as Stalnaker and Lewis, believed that certain apparent counterexamples to Antecedent Strengthening—now known as *Sobel Sequences*—refuted SA.¹ More recently, defenders of SA have responded by enriching SA with certain *dynamic* principles governing how context evolves. They argue that Sobel sequences are not counterexamples to a *Dynamic Strict Analysis* (Dynamic SA).

But Antecedent Strengthening is just one of a family of strengthening principles. In this paper, we focus on a weaker principle, which we call *Strengthening with a Possibility*, and give a counterexample to it. We

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¹ Robert C. Stalnaker, "A Theory of Conditionals," in Nicholas Rescher, ed., *Studies in Logical Theory* (Oxford: Blackwell, 1968), pp. 98–112; and David Lewis, *Counterfactuals* (Oxford: Blackwell, 1973).

show that the dynamic features attributed to *would*-counterfactuals and *might*-counterfactuals by proponents of Dynamic SA are of no help when it comes to counterexamples to Strengthening with a Possibility, unlike counterexamples to Antecedent Strengthening itself.

We develop a new version of VSA that incorporates a Kratzerian *ordering source* into the meaning of counterfactuals, and we show how to model counterexamples to Strengthening with a Possibility on our account. In the final section, we address a worry about our account concerning counterfactuals with disjunctive antecedents.

I. TWO THEORIES OF COUNTERFACTUALS

Both VSA and SA assume that context supplies a comparative closeness ordering on worlds, represented by $\preceq_{c,w}$, which compares any two worlds with respect to their similarity to a benchmark world w .² We assume that $\preceq_{c,w}$ is a comparative *similarity* relation, so that $w_1 \preceq_{c,w} w_2$ means that w_1 is at least as similar to w as w_2 is.

VSA uses a *selection function*, a contextually determined function f_{\preceq_c} that takes an antecedent A , and a world w , and returns the set of closest A -worlds to w , according to $\preceq_{c,w}$. A world w_1 is among the closest A -worlds to w just if there is no other A -world w_2 that is closer to w than w_1 is. Where $A \Box \rightarrow C$ stands for the counterfactual conditional with antecedent A and consequent C , VSA's semantic entry for the counterfactual runs as follows:

- (2) *Variably Strict Account (VSA)* $\llbracket A \Box \rightarrow C \rrbracket^{c,w} = 1$ iff $\forall w' \in f_{\preceq_c}(A, w) : \llbracket C \rrbracket^{c,w'} = 1$.
 $A \Box \rightarrow C$ is true relative to a context c and world w just if, for every world w' if w' is among the closest A -worlds to w , then C is true in w' .³

SA replaces the selection function with a contextually determined accessibility relation \min_{\preceq_c} that takes a world w and returns the set of

² $\preceq_{c,w}$ is transitive (for all w_1, w_2, w_3 if $w_1 \preceq_{c,w} w_2$, and $w_2 \preceq_{c,w} w_3$, then $w_1 \preceq_{c,w} w_3$), reflexive (for all $w_1, w_1 \preceq_{c,w} w_1$), and antisymmetric (for all w_1, w_2 , if $w_1 \preceq_{c,w} w_2$ and $w_2 \preceq_{c,w} w_1$, then $w_1 = w_2$). It is also at least *weakly centered*: there is no w_1 such that $w_1 \prec_{c,w} w$.

³ This statement of VSA makes the *limit assumption*: for all worlds w and propositions A , there is at least one closest A -world to w . This is purely for ease of presentation; VSA can be stated without making the limit assumption, but it is unnecessarily complicated for our purposes. Our statement of VSA is neutral about the *uniqueness assumption*: for all w and A , there is a unique closest A -world to w . See Stalnaker, "A Theory of Conditionals," *op. cit.*; and Robert C. Stalnaker, "A Defense of Conditional Excluded Middle," in William Harper, Robert Stalnaker, and Glenn Pearce, eds., *Ifs* (Dordrecht, The Netherlands: D. Reidel, 1981), pp. 87–104, for defenses of both the limit assumption and the uniqueness assumption. See Lewis, *Counterfactuals*, *op. cit.*, for arguments against both principles.

closest worlds to w according to $\preceq_{c,w}$. (Unlike the selection function f_{\preceq_c} , \min_{\preceq_c} does not take the antecedent as argument.) Here is the semantic entry given by SA:

- (3) *Strict Account (SA)*. $\llbracket A \Box \rightarrow C \rrbracket^{c,w} = 1$ iff $\forall w' \in \min_{\preceq_c}(w) \cap \llbracket A \rrbracket^c$:
 $\llbracket C \rrbracket^{c,w'} = 1$.
 $A \Box \rightarrow C$ is true relative to a context c and world w just if, for every world w' , if w' is among the closest worlds to w , and A is true in w' , then C is true in w' .

The difference between SA and VSA is significant—whether the evaluation domain is variable or fixed matters to the *logic* of the counterfactual. Here is a principle VSA and SA disagree about:

- (4) *Antecedent Strengthening*. $A \Box \rightarrow C \models (A \wedge B) \Box \rightarrow C$

SA validates Antecedent Strengthening, whereas VSA does not. SA does not allow the evaluation domain to vary. If all the closest worlds where A is true are worlds where C is true, then *a fortiori*, all of the closest worlds where A and B are true are worlds where C is true. On the other hand, VSA allows the evaluation domain to vary from antecedent to antecedent: The closest AB -worlds need not be the closest A -worlds. So, what is true in all of the closest A -worlds may be false in some of the closest AB -worlds.

II. SOBEL SEQUENCES AND DYNAMIC SA

Antecedent Strengthening seems subject to counterexample. Consider the *Sobel sequence* (5) below:

- (5) a. If I had struck the match, it would have lit.
 b. But of course, if I had struck the match *and* it had been soaked in water last night, it would not have lit.

Sentences (5-a) and (5-b) seem consistent. Indeed, in most ordinary match-striking scenarios, (5-a) and (5-b) are both true. Typically, when you strike a match, the match is dry and lights easily. Soaking wet matches are not as cooperative—if you strike a match drenched in water, it does not light.

But if Antecedent Strengthening is valid, Sobel sequences like (5) are not consistent. If it is true that if I had struck the match, it would have lit, then (by Antecedent Strengthening) it follows that if I had struck the match *and* the match had been soaked in water, the match would (still) have lit.

That it validates Antecedent Strengthening would seem to be a clear strike against SA. But it turns out there is a lot more to be said

for SA here. As von Fintel shows, a suitably sophisticated strict conditional analysis *can* account for the sequence in (5).⁴ His strategy is to appeal to the dynamic effects of counterfactuals in conversation: Though (5-b) is not true when (5-a) is uttered, *asserting* (5-b) changes the context so that it comes out true.

Why would that be so? Von Fintel proposes that counterfactuals presuppose that their domains contain some worlds where the antecedent is true. When that presupposition is not met, the context is minimally altered to ensure that it is. Suppose a speaker utters a counterfactual $A \Box \rightarrow C$. If the domain contains A -worlds, nothing changes; if it does not, it *expands* to include the closest A -worlds. $A \Box \rightarrow C$ is true in this *new* context just in case all of the A -worlds in the expanded set are C -worlds.

Let us apply von Fintel's *dynamic strict account* (Dynamic SA) to our example. When the speaker asserts (5-a), the domain expands to include the closest worlds where she strikes the match. (5-a) is true. So in all of these worlds, the match lights. But any world where the match lights is one where the match is *dry*. So the presupposition of (5-b) is not satisfied. When the speaker *utters* (5-b), the context changes to accommodate the presupposition—the domain expands to include the closest worlds where she strikes the match and the match is soaking wet. Since all of these worlds are ones where the match does not light, (5-b) comes out true.

Now that we have an informal understanding of how Dynamic SA works we can state the view more precisely.⁵ Let h_c be a contextually supplied accessibility relation. Then:

- (6) *Dynamic Strict Account (Dynamic SA)*.
- a. $\llbracket A \Box \rightarrow C \rrbracket^{c,w}$ is defined only if $h_c(w) \cap \llbracket A \rrbracket^c \neq \emptyset$
 - b. Where $\llbracket A \Box \rightarrow C \rrbracket^{c,w}$ is defined, $\llbracket A \Box \rightarrow C \rrbracket^{c,w} = 1$ iff $\forall w' \in h_c(w) \cap \llbracket A \rrbracket^c: \llbracket C \rrbracket^{c,w'} = 1$.

There are two parts to Dynamic SA: *definedness conditions* and *truth-conditions*. The definedness conditions, which are stated in (6-a), take the form of a semantic presupposition about the contextually supplied accessibility relation. The presupposition is a *compatibility presupposition*: the accessibility relation h_c must assign to the world of evalua-

⁴Kai von Fintel, "Counterfactuals in a Dynamic Context," in Michael J. Kenstowicz, ed., *Ken Hale: A Life in Language* (Cambridge, MA: MIT Press, 2001), pp. 123–52.

⁵Note that von Fintel adds the dynamic effects directly into the semantics, building a dynamic context-change potential into the semantic entry for the counterfactual. This version of the view is more complicated to state, so we prefer to work with the formulation of Dynamic SA that puts the dynamic effects in the pragmatics. Nothing we say about Dynamic SA hangs on this choice.

tion w at least some worlds where A is true. In other words, $h_c(w)$ must be compatible with A . If $h_c(w)$ is not compatible with A , $\llbracket A \Box \rightarrow C \rrbracket^{c,w}$ is undefined. The truth-conditions for the counterfactual are stated in (6-b). They tell us when the counterfactual is true if it is defined. Specifically, they say that if $\llbracket A \Box \rightarrow C \rrbracket^{c,w}$ is defined, then $\llbracket A \Box \rightarrow C \rrbracket^{c,w}$ is true just in case the following holds: for every world w' , if w' is among the closest worlds to w , and A is true in w' , then C is true in w' .

This leaves open two important questions. First, what is the identity of the initial accessibility relation h_c ? And second, what happens when $\llbracket A \Box \rightarrow C \rrbracket^{c,w}$ is undefined—that is, when h_c does not assign to the world of evaluation any worlds where A is true? We said that when the compatibility presupposition of the counterfactual is not met, the context is minimally altered to ensure that it is. But what, exactly, does that mean?

Von Fintel provides answers to both questions. He assumes the initial accessibility relation assigns to any evaluation world w the singleton set of that world itself $\{w\}$. His answer to the second question is more detailed, but the basic idea is that when a counterfactual $A \Box \rightarrow C$ is undefined relative to a context c and world w , asserting the counterfactual updates the context by adding to the counterfactual domain *all of the closest A -worlds to w* according to \preceq_c .⁶ The counterfactual is true in this updated context just in case all of the A -worlds in the expanded domain are C -worlds.

It is important to note that Antecedent Strengthening is not *classically* valid on Dynamic SA. An inference is classically valid just in case its conclusion is true whenever its premises are. Dynamic SA says that Antecedent Strengthening is merely *Strawson valid*: whenever $A \Box \rightarrow C$ is true, and $(A \wedge B) \Box \rightarrow C$ is *defined*, $(A \wedge B) \Box \rightarrow C$ is true too.⁷ Dynamic SA allows contexts where $A \Box \rightarrow C$ is true yet $(A \wedge B) \Box \rightarrow C$ is undefined. (This will happen whenever the domain contains A -worlds, but no B -worlds, and all of the A -worlds are C -worlds.) This is critical to its account of Sobel sequences. It is the fact that (5-b) is undefined (rather than simply false) that forces the context to change when (5-b) is asserted so that (5-b) comes out true.

We have seen that Sobel sequences like (5) do not refute Dynamic SA, even if they refute the simplest strict analysis. But von Fintel goes

⁶ Cory Nichols objects to this feature of Dynamic SA and furthermore argues that any plausible way of accommodating the presupposition leads to unwelcome consequences; see Cory Nichols, "Strict Conditional Accounts of Counterfactuals," *Linguistics and Philosophy*, xl, 6 (December 2017): 621–45. Our problem is separate from those he discusses, however.

⁷ More generally, the inference from $\llbracket A \rrbracket^c, \llbracket P_1 \rrbracket^c, \dots, \llbracket P_n \rrbracket^c$ to $\llbracket C \rrbracket^c$ is Strawson-valid iff for any w such that $\llbracket A \rrbracket^{c,w}, \llbracket P_1 \rrbracket^{c,w}, \dots, \llbracket P_n \rrbracket^{c,w}$ and $\llbracket C \rrbracket^{c,w}$ are all defined and such that $\llbracket A \rrbracket^{c,w} = \llbracket P_1 \rrbracket^{c,w} = \dots = \llbracket P_n \rrbracket^{c,w} = 1, \llbracket C \rrbracket^{c,w} = 1$ also.

further, arguing we have reason to *prefer* Dynamic SA to VSA. He observes that when we reverse the order of (5-a) and (5-b), the sequence sounds much worse:

- (7) a. If I had struck the match and it had been soaked in water last night, it would not have lit.
 b. But if I had struck the match, it would have lit.

VSA predicts no difference between (5) and (7): *both* sequences should sound fine. (5-a) and (5-b) are consistent; reversing the order cannot change that.

Dynamic SA, on the other hand, has a persuasive account of the infelicity of (7). That account runs something like this. Find the closest worlds to actuality, according to the standards of similarity in the context. Among those worlds, identify the ones where the speaker strikes the match and it is wet. (7-a) says that all of these are worlds where the match does not light; (7-b) says that all of them are worlds where the match does light. No wonder that (7) is infelicitous—(7-a) and (7-b) contradict each other.

Some VSA theorists, such as Sarah Moss, have argued in response that what Dynamic SA explains *semantically*, VSA can explain by appeal to warranted assertability.⁸ Though semantically consistent, Moss argues, reverse Sobel sequences are pragmatically inappropriate.

We think the debate has reached something of a stalemate. We have two explanations of the same data—one pragmatic, one semantic. Neither has a clear empirical edge.⁹ We move to change the debate. Antecedent Strengthening is just one of a family of strengthening principles—and indeed, it is the strongest member of that family. By Strawson-validating Antecedent Strengthening, Dynamic SA predicts that a whole host of strengthening principles are Strawson-valid, too. We argue that this prediction is unwelcome. We focus on one strengthening principle—which we call *Strengthening with a Possibility*—and present a counterexample to this principle. Furthermore,

⁸ See Sarah Moss, “On the Pragmatics of Counterfactuals,” *Noûs*, XLVI, 3 (September 2012): 561–86. Moss is not the only one who has defended VSA against reverse Sobel sequences. See also Karen Lewis, “Counterfactual Discourse in Context,” *Noûs*, LII, 4 (September 2018): 481–507, for an alternative story within the VSA framework, according to which raising skeptical possibilities to salience renders examples like (7-b) false.

⁹ Proponents of VSA might point out that VSA has an easier time predicting the felicitous reverse Sobel sequences noted by Moss, “On the Pragmatics of Counterfactuals,” *op. cit.* But, on the other hand, proponents of SA might point out that their theory has an easier time explaining why Negative Polarity Items (NPIs) are licensed in the antecedents of counterfactuals (as noted in von Stechow, “Counterfactuals in a Dynamic Context,” *op. cit.*). Thus, while these theories are clearly not empirically equivalent, we think that the literature so far has not shown that one theory has a clear edge.

we show that Dynamic SA *classically* validates this principle, rather than (merely) Strawson-validating it. This is significant, for it means that Dynamic SA's dynamic resources are of no help when it comes to counterexamples to Strengthening with a Possibility.

III. STRENGTHENING WITH A POSSIBILITY

We can think of a strengthening principle as a principle that allows us to move from a counterfactual $A \Box \rightarrow C$, along with certain auxiliary premises, to a counterfactual with a strengthened antecedent $(A \wedge B) \Box \rightarrow C$. More formally, where $n \geq 0$, we have:

$$(8) \textit{ Strengthening Principle. } A \Box \rightarrow C, P_1, \dots, P_n \vDash (A \wedge B) \Box \rightarrow C$$

Antecedent Strengthening is the instance of (8) where $n = 0$. It says that *no* further premises are needed to strengthen the antecedent of a counterfactual. This makes it the strongest strengthening principle: a semantics that validates it validates *every* strengthening principle. Classical validity is monotonic: *adding* premises never turns a valid inference into an invalid one. If $A \Box \rightarrow C$ entails $(A \wedge B) \Box \rightarrow C$ *all by itself*, $A \Box \rightarrow C$ still entails $(A \wedge B) \Box \rightarrow C$ when combined with other premises. Similar reasoning shows that *Strawson-validating* Antecedent Strengthening Strawson-validates every other strengthening principle—Strawson-entailment is also monotonic.¹⁰

But there are weaker strengthening principles. These principles allow us to strengthen the antecedent of a counterfactual not with just *any* conjunct, but only those that satisfy some auxiliary premises. We are interested in one of these weaker principles, which we call *Strengthening with a Possibility*.

Before we state the principle, a few preliminary remarks are in order.

III.1. 'Might' Counterfactuals. So far we have focused on English *would*-counterfactuals like (1):

- (1) If David's alarm had not gone off this morning, he would have missed class.

But *would*-counterfactuals are not the only kind of counterfactual. There are also *might*-counterfactuals. For example:

- (9) If David's alarm had not gone off this morning, he might have missed class.

¹⁰ *Proof:* Suppose that $\llbracket A \rrbracket^c, \llbracket P_1 \rrbracket^c, \dots, \llbracket P_n \rrbracket^c \not\vDash_{Str} \llbracket C \rrbracket^c$. Then there must be some w such that $\llbracket A \rrbracket^{c,w} = \llbracket P_1 \rrbracket^{c,w} = \dots = \llbracket P_n \rrbracket^{c,w} = 1$ but $\llbracket C \rrbracket^{c,w} = 0$. But then, since w itself is a world where $\llbracket A \rrbracket^{c,w} = 1$ but $\llbracket C \rrbracket^{c,w} = 0$, we have $\llbracket A \rrbracket^c \not\vDash_{Str} \llbracket C \rrbracket^c$. Contraposing, if $\llbracket A \rrbracket^c \vDash_{Str} \llbracket C \rrbracket^c$ then $\llbracket A \rrbracket^c, \llbracket P_1 \rrbracket^c, \dots, \llbracket P_n \rrbracket^c \vDash_{Str} \llbracket C \rrbracket^c$.

We need to say what *might*-counterfactuals like (9) mean.

We will write $A \diamondrightarrow B$ for the *might*-counterfactual with antecedent A and consequent C . We will assume that $A \diamondrightarrow B$ is the *dual* of $A \squarerightarrow B$. What does that mean? The dual of an operator O is a term which has the meaning of $\neg O\neg$. To say that $A \diamondrightarrow B$ is the dual of $A \squarerightarrow B$ is to say that $A \diamondrightarrow B$ has the same meaning as $\neg(A \squarerightarrow \neg B)$. For instance, (9) has the same meaning as (10):

- (10) It is not the case that if David's alarm had not gone off this morning, he *would not* have missed class.

The assumption that English *might*-counterfactuals are the duals of *would*-counterfactuals is called *Duality*.¹¹ Note that Duality is controversial: many deny it, including various defenders of Counterfactual Excluded Middle.¹² We assume Duality merely for ease of exposition, as our central counterexample can be stated without it (see footnote 14 for further details).

If we assume Duality, then we can state the semantics for *might*-counterfactuals given by Dynamic SA as follows:

- (11) *Dynamic SA* ($A \diamondrightarrow C$)
- a. $\llbracket A \diamondrightarrow C \rrbracket^{c,w}$ is defined only if $h_c(w) \cap \llbracket A \rrbracket^c \neq \emptyset$
 - b. Where $\llbracket A \diamondrightarrow C \rrbracket^{c,w}$ is defined, $\llbracket A \diamondrightarrow C \rrbracket^{c,w} = 1$ iff $\exists w' \in h_c(w) \cap \llbracket A \rrbracket^c: \llbracket C \rrbracket^{c,w'} = 1$.

The semantics for \diamondrightarrow has two parts, just as it does for \squarerightarrow . First we have the definedness condition stated in (11-a): $A \diamondrightarrow C$ is defined only if h_c assigns to the world of evaluation at least some worlds where A is true (this part is the same for \diamondrightarrow as it is for \squarerightarrow). And second we have the truth-conditions which are set down in (11-b): where $\llbracket A \diamondrightarrow C \rrbracket^{c,w}$ is defined, $\llbracket A \diamondrightarrow C \rrbracket^{c,w}$ is true just in case there is some world w' such that w' is among the closest worlds to w , A is true in w' , and C is true in w' .

What does VSA say about *might*-counterfactuals? Assuming Duality, VSA gives \diamondrightarrow the following semantics:

¹¹ Note that von Stechow seems to accept Duality, as does Anthony S. Gillies, "Counterfactual Scorekeeping," *Linguistics and Philosophy*, xxx, 3 (November 2007): 329–60, another proponent of Dynamic SA. Moreover, Duality falls out of the widely accepted restrictor analysis of conditionals in Angelika Kratzer, "Conditionals," *Chicago Linguistics Society*, xxii, 2 (1986): 1–15, where the 'might' will only quantify over worlds that make the antecedent true, and so *might*-counterfactuals will have the truth-conditions of \diamondrightarrow .

¹² See, among others, Stalnaker, "A Defense of Conditional Excluded Middle," *op. cit.*; and J. Robert G. Williams, "Defending Conditional Excluded Middle," *Noûs*, XLIV, 4 (December 2010): 650–68, for arguments against Duality.

$$(12) \text{ VSA } (A \diamondrightarrow C). \llbracket A \diamondrightarrow C \rrbracket^{c,w} = 1 \text{ iff } \exists w' \in f_{\leq c}(A, w) : \llbracket C \rrbracket^{c,w'} = 1.$$

VSA says that $A \diamondrightarrow C$ is true relative to a context c and world w just if there is some world w' that is among the closest A -worlds to w that is also a C -world.

III.2. Strengthening with a Possibility and Dynamic SA. We are now in a position to state the strengthening principle that we are interested in.

$$(13) \text{ Strengthening with a Possibility. } (A \squarerightarrow C) \wedge (A \diamondrightarrow B) \models (A \wedge B) \squarerightarrow C$$

(13) says that one can strengthen an antecedent with any proposition with which that antecedent is *counterfactually consistent*. Take an example. Suppose it is true that if I had taken modal logic next semester, I would have passed. Does that mean that I would have passed had I taken the class and the class was taught by Joe? According to Strengthening with a Possibility, that depends on whether Joe *might* have been the teacher, had I taken the class. If Joe could not have taught the class—say, because he was on leave—I can truly say that I would have passed even if I would have bombed a class taught by Joe. On the other hand, if Joe might have taught the class, then I cannot truly say that I would have passed unless I would have passed Joe's class, too.

We said that Antecedent Strengthening is the strongest strengthening principle. So, by Strawson-validating Antecedent Strengthening, Dynamic SA Strawson-validates Strengthening with a Possibility. But we can show something stronger: by Strawson-validating Antecedent Strengthening, Dynamic SA *classically* validates Strengthening with a Possibility. The proof is straightforward. Suppose that for a given context c , (i) $A \squarerightarrow C$ is true in c , and (ii) $A \diamondrightarrow B$ is true in c . It follows from (ii) and Dynamic SA that the domain in c contains worlds where A and B are both true. But that is just to say (iii) that $(A \wedge B) \squarerightarrow C$ is *defined* in c . Since Antecedent Strengthening is Strawson-valid, (i)—that $A \squarerightarrow C$ is true in c —and (iii)—that $(A \wedge B) \squarerightarrow C$ is defined in c —together entail that $(A \wedge B) \squarerightarrow C$ is true in c .

This fact is important. If Strengthening with a Possibility is classically valid, we cannot appeal to the dynamic resources of Dynamic SA to account for apparent counterexamples. To see this, return to the match example:

- (5) a. If I had struck the match, it would have lit.
- b. But of course, if I had struck the match *and* it had been soaked in water last night, it would not have lit.

Earlier we noted that Dynamic SA Strawson-validates but does not classically validate Antecedent Strengthening. Dynamic SA does not say

that (5-b) is true in any context in which (5-a) is true—it allows contexts where (5-a) is true yet (5-b) is *undefined*. That is how it accounts for the felicity of Sobel sequences like (5). If (5-b) is undefined when (5-a) is uttered, asserting (5-b) shifts the context so that it comes out true.

If Strengthening with a Possibility is classically valid, Dynamic SA cannot account for apparent counterexamples to Strengthening with a Possibility in the same way. By the definition of classical validity, $(A \wedge B) \Box \rightarrow C$ is *defined* (and true) in any context in which $A \Box \rightarrow C$ and $A \Diamond \rightarrow B$ are true. But if $(A \wedge B) \Box \rightarrow C$ is defined, then *asserting* $(A \wedge B) \Box \rightarrow C$ will not change the context. The domain will not expand to make $(A \wedge B) \Box \rightarrow C$ false, as we would hope; $(A \wedge B) \Box \rightarrow C$ will simply come out true in the original context in which $A \Box \rightarrow C$ and $A \Diamond \rightarrow B$ are uttered.

Dynamic SA is then banking on the (classical) validity of Strengthening with a Possibility. It wagers that $A \Box \rightarrow C$ and $A \Diamond \rightarrow B$ together entail $(A \wedge B) \Box \rightarrow C$. In the next section, we show that Strengthening with a Possibility is not valid—it has counterexamples. Dynamic SA rests on a mistake.

IV. A COUNTEREXAMPLE TO DYNAMIC SA

Consider this case:¹³

Dice: Alice, Billy, and Carol are playing a simple game of dice. Anyone who gets an odd number wins \$10; anyone who gets even loses \$10. The die rolls are, of course, independent. What Alice rolls has no effect on what Billy rolls and vice versa. Likewise for Alice and Carol as well as for Billy and Carol.

Each player throws their dice. Alice gets odd; Billy gets even; Carol gets odd.

Now consider this sequence of counterfactuals:

- (14) a. If Alice and Billy had thrown the same type of number, then at least one person would still have won \$10.
 b. If Alice and Billy had thrown the same type of number, then Alice, Billy, and Carol could have *all* thrown the same type of number. (Because they could have all thrown odd.)
 c. If Alice, Billy, and Carol had all thrown the same type of number, then at least one person would still have won \$10.

¹³This case was inspired by one used in Robert Stalnaker, "What Is a Nonmonotonic Consequence Relation?," *Fundamenta Informaticae*, xxi, 1 (April 1994): 7–21, to argue against certain principles of belief revision.

(14-a) and (14-b) seem true, but (14-c) is dubious. (14-a) seems right because if Alice and Billy had thrown the same type of number, nothing would have changed with respect to *Carol*—she would still have rolled odd. So someone would still have won \$10.

(14-b) seems right too. If Alice and Billy had thrown the same type of number, either Alice or Billy would have gotten a different number from the one they actually got. But there is no reason to think it would have been Alice rather than Billy: Billy might have thrown odd, along with Alice and Carol.

But (14-c) seems wrong. There are two ways for Alice, Billy, and Carol to throw the same type of number. They could all roll odd or they could all roll even. And we cannot just rule out the latter. If Alice, Billy, and Carol had thrown the same type of number, they might have thrown even, so there might have been no winner: (14-c) is false.

Dynamic SA wrongly predicts that (14-c) follows from (14-a) and (14-b). For (14-a), (14-b), and (14-c) are respectively equivalent to:¹⁴

- (15) a. *Alice Billy same* $\Box \rightarrow$ *someone wins \$10*
 b. *Alice Billy same* $\Diamond \rightarrow$ (*Alice Billy same* \wedge *Billy Carol same*)
 c. (*Alice Billy same* \wedge *Billy Carol same*) $\Box \rightarrow$ *someone wins \$10*

Suppose (15-a) and (15-b) are true. Since (15-b) is true, some worlds in the domain are ones where its antecedent and consequent are true—that is, where Alice, Billy, and Carol all throw the same type of number. But that is just to say that (15-c) is *defined*. Dynamic SA Strawson-validates Antecedent Strengthening. So, if (15-a) is true, and (15-c) is defined, then (15-c) must be true, too. This is the wrong result. (15-a) and (15-b) are true, and (15-c) is not.

Dice is a very simple example with this structure, and so it will be our focus for the rest of the paper. However, note that there are examples with this structure that are less artificial and that do not involve chance. Consider this example:

¹⁴In assuming that (14-b) is equivalent to (14-b)*, we assume Duality. However, as we noted, the counterexample does not ultimately rely upon it. We can state the dual of the *would*-counterfactual using wide-scope negation:

- (i) a. If Alice and Billy had thrown the same type of number, then at least one person would still have won \$10.
 b. It is not true that if Alice and Billy had thrown the same type of number, then Alice, Billy, and Carol *would not* have all thrown the same type of number.
 c. If Alice, Billy, and Carol had all thrown the same type of number, then at least one person would still won \$10.

We notice no difference in our judgments here.

The Party. Alice, Billy, and Carol have all been invited to parties at Xander's and at Yulia's. Xander's party will be wild and Yulia's will be relaxed, and all are choosing purely on the basis of which experience they would rather have tonight. (They do not know each other and their preferences are totally independent of each other, so where one goes will not at all determine where the others go.) As a matter of fact Alice and Carol go to Xander's party and Billy goes to Yulia's.

Now consider the following sequence of counterfactuals:

- (16) a. If Alice and Billy had gone to the same party, then at least one of the three would (still) have been at Xander's.
 b. If Alice and Billy had gone to the same party, then all three might have all gone to the same party.
 c. If all three had gone to the same party, then at least one of the three would have been at Xander's.

We judge that (16-a) and (16-b) are true, whereas (16-c) is not true. Dynamic SA cannot make these predictions. If (16-b) is true, then (16-c) is defined. But if (16-c) is defined, then (16-a) is true only if (16-c) is true. In other words, for Dynamic SA, (16-a) and (16-b) entail (16-c).

V. RESCUE ATTEMPTS

In its current form, Dynamic SA cannot account for our judgments about these sentences. In this section, we explore whether there is a way to modify it that does better. To get clear on what the options are, we return to *Dice*. We know that someone wins if and only if someone rolls odd. To predict that (14-a) is true, there must be someone who rolls odd in all domain worlds where Alice and Billy roll the same type of number. And to predict that (14-c) is false, some domain worlds where Alice and Billy (and Carol) roll the same type of number must be ones where everyone rolls *even*. The domain must expand between utterances of (14-a) and (14-c). It must start out containing no worlds where Alice, Billy, and Carol all throw even, and acquire some by the time we evaluate (14-c).

There are only two ways for the domain to expand between (14-a) and (14-c). Either asserting (14-b) expands the domain, or asserting (14-c) does. We already saw that standard Dynamic SA does not allow asserting (14-c) to expand the domain. We will first explore a proposal about *might*-counterfactuals, due to Gillies,¹⁵ which makes (14-b) expand the domain. Then we will explore an alternative suggestion, due to an anonymous referee, that makes (14-c) expand the domain. We conclude neither is successful.

¹⁵ Gillies, "Counterfactual Scorekeeping," *op. cit.*

V.I. *Dynamic ‘Might’s to the Rescue?* We have been assuming that *might*-counterfactuals update the domain just like *would*-counterfactuals do, presupposing that their antecedents are possible with respect to the counterfactual domain. But this account will not allow (14b) to shift the domain. (14a) and (14b) have the same antecedent, so there can be no shifting that is triggered by the latter that is not already triggered by the former. We know that (14a) does not expand the domain to include worlds where Alice, Billy, and Carol throw even—if it did, (14a) would come out *false*, but we hear it as true. So asserting (14b) cannot trigger a domain expansion either.

Gillies offers a different idea.¹⁶ He says that the dynamic effect of *might*-counterfactuals is distinct from that of *would*-counterfactuals. $A \diamond \rightarrow B$ presupposes that the domain contains worlds where A and B are both true.¹⁷ For example, (14b) presupposes that the domain contains worlds where Alice, Billy, and Carol all roll the same.¹⁸

Here is how things would have to go if Gillies’s theory is to help with our case. For simplicity, suppose that the initial context in *Dice* is such that, in all domain worlds, everyone rolls as they actually do—Alice and Carol roll odd, and Billy rolls even. (14a)’s presupposition is not met in this context, so asserting (14a) expands the domain, adding worlds where Alice and Billy roll the same. Suppose we include worlds where Alice and Billy roll even, but none where they roll odd. We cannot include any worlds where *Carol* rolls even, lest we render (14a) false. But if we add no worlds where Alice and Billy

¹⁶ *Ibid.*

¹⁷ Note that at the end of the day, Gillies does not think this assertability condition is a genuine presupposition. While he calls it an entertainability presupposition, he rightly points out that the presupposition of the *might*-counterfactual is not plausibly an existence presupposition on a quantifier domain. We do not rely on any particular way of cashing out entertainability presuppositions in our arguments.

¹⁸ As Gillies himself notes, this has an unintuitive consequence in a static framework: it makes all utterances of $A \diamond \rightarrow B$ true. Nonetheless, as Gillies points out, it looks like Dynamic SA needs to help itself to something like this kind of effect because *might*-counterfactuals seem to give rise to sequences similar to Sobel sequences. Compare:

- (i) a. If Sophie went to the parade, she would have seen Pedro.
b. But if she had gone and been stuck behind someone tall, she might not have seen him.
- (ii) a. If Sophie had gone to the parade and been stuck behind someone tall, she might not have seen Pedro.
b. # But if she had gone to the parade, she would have seen Pedro.

Without attributing some dynamic effects to *might*-counterfactuals, these sequences would be counterexamples to Dynamic SA, just as our original Sobel sequences were counterexamples to the simplest form of SA. Gillies chooses to retain this dynamic effect and discusses some ways of addressing the problems it gives rise to.

roll odd, and no worlds where Carol rolls even, the domain will not contain any worlds where Alice, Billy, and Carol all roll the same type of number, and so the presupposition of (14-b) will not be met. (The consequent of (14-b) is true in a world only if Alice, Billy, and Carol roll the same there.) This means that asserting (14-b) will add worlds where Alice, Billy, and Carol all throw the same type of number. If we include worlds where they all throw even, (14-c) comes out false.

One problem is that our Gillies-inspired story cannot be told when (14-b) is uttered *before* (14-a), as in the following sequence:

- (14-b) If Alice and Billy had thrown the same type of number, then Alice, Billy, and Carol could have *all* thrown the same type of number (because Alice and Billy might have thrown odd along with Carol).
- (14-a) If Alice and Billy had thrown the same type of number, then someone would still have won \$10.
- (14-c) If Alice, Billy, and Carol had all thrown the same type of number, then at least one person would still have won \$10.

Reversing the order of (14-a) and (14-b) does not change our judgments. (14-a) still seems true: if Alice and Billy had rolled the same type of number, *Carol* would still have rolled odd, so someone would still have won \$10. And (14-c) still seems false: If Alice, Billy, *and* Carol had rolled the same type of number, they might have all rolled even, in which case *nobody* would have won. The problem is that if (14-b) introduces worlds where Alice, Billy, and Carol all throw even, (14-a) will come out false. This is the wrong prediction. Even when uttered after (14-b), (14-a) seems true.

There are also problems when (14) is uttered in its original order. Suppose (14-b) introduces worlds where Alice, Billy, and Carol all throw even. This will indeed make (14-c) false. But it has other, less welcome consequences. Consider the sequence:

- (14-b) If Alice and Billy had rolled the same type of number, Alice, Billy, and Carol might have *all* rolled the same type of number (because Alice and Billy might have thrown odd along with Carol).
- (17) If Alice and Billy had rolled the same type of number, Carol might not have rolled odd.

(14-b) is true, but (17) is false. Indeed, (17) is false for the same reason that (14-a) is true—there is no reason to suppose that, if Alice and Billy had rolled the same type of number, things might have changed with respect to *Carol*. She would have still rolled odd. But if

(14b) adds to the domain worlds where Alice, Billy, and Carol throw even, (17) will come out true, contrary to intuition.¹⁹

We do not want (14b) to add worlds where Alice, Billy, and Carol all roll even. When we evaluate (14b), we continue to hold fixed that Carol rolls odd—that is why we judge (17) false. (We judge (14b) true not because we think they might have all thrown even, but because we think they might have all thrown odd.) To be sure, things change by the time we get to (14c). At that point, we are considering worlds where they all throw even—we judge (14c) false because they might have all thrown even and lost. But it is not (14b) that makes those worlds relevant. It is only when we hear (14c) that we consider worlds where Carol rolls even.

Let us take stock. Our goal was to predict that (14a) and (14b) are true, and that (14c) is false. We said that Dynamic SA classically validates Strengthening with a Possibility. Any context in which (14a) and (14b) are true is one in which (14c) is true. This was an important observation. It meant that we could not appeal to the dynamic effects of (14c) to account for our false judgment of this sentence: If (14c) is true, it is also defined, so asserting (14c) will not expand the domain. We then asked whether asserting (14b) could expand the domain to make (14c) false. We found that it could, but this offered little comfort. For one, if we reverse the order of (14a) and (14b), we wrongly predict that (14a) is false. For another, if asserting (14b) adds worlds where Alice, Billy, and Carol roll even, we will indeed predict that (14c) is false, but we will *also* predict that (17) is true. But (17) is heard as false.

V.2. *(In)dependence to the Rescue?* Our judgments about counterfactuals tend to respect causal independence: if some fact *B* is causally independent of whether *A* holds, we typically hold it fixed when evaluating counterfactuals of the form $A \square \rightarrow B$.²⁰ As an anonymous referee correctly notes, causal independence plays an important role in

¹⁹We can make this same point with the following *would*-counterfactual:

- (i) If Alice and Billy had rolled the same type of number, Carol would still have rolled odd.

(i) is intuitively true. But if (14b) introduces worlds where Alice, Billy, and Carol roll even, (i) will come out false.

²⁰Something like this thought goes at least as far back as Nelson Goodman, “The Problem of Counterfactual Conditionals,” this JOURNAL, XLIV, 5 (February 1947): 113–28, and can be seen in Kratzer’s influential *premise semantics* for counterfactuals. See Rachael A. Briggs, “Interventionist Counterfactuals,” *Philosophical Studies*, CLX, 1 (August 2012): 139–66; and Stefan Kaufmann, “Causal Premise Semantics,” *Cognitive Science*, XXXVII (August 2013): 1136–70, for detailed discussion of the relationship between counterfactuals and causal dependence.

our example: (14a) and (14b) seem true because changing the facts about Alice and Billy's rolls should not change what happens with Carol's roll. On the other hand, (14c) seems false because whether Carol rolls odd is not independent of what Alice, Billy, and Carol roll.

As we presented the view, Dynamic SA does not say anything about how facts about causal independence affect the evolution of the counterfactual domain. But perhaps this is a mistake; it is undeniable that our judgments about counterfactuals are heavily influenced by our beliefs about causal dependencies. In the dynamic framework, a way to capture this is to make the presuppositions of counterfactuals sensitive to facts about causal independence. Once we make this change, perhaps Dynamic SA can predict that the counterfactual domain expands between (14b) and (14c): the antecedent of (14b) is causally independent of Carol's roll, but the antecedent of (14c) is not.

A natural move is to add an *independence presupposition* to Dynamic SA. In addition to presupposing that there are some antecedent worlds in the domain, a counterfactual $A \square \rightarrow B$ presupposes that, for any C that is true in the world of evaluation and causally independent of A , C is true throughout the domain.

- (18) *Independence Presupposition.* $\llbracket A \square \rightarrow C \rrbracket^{c,w}$ is defined only if $(h_c(w) \cap \llbracket A \rrbracket^c) \subseteq \llbracket B \rrbracket^c$, for any B such that: (i) $\llbracket B \rrbracket^{c,w} = 1$ and (ii) B is causally independent of A .

We think that proponents of Dynamic SA should not accept the Independence Presupposition as stated. Doing so would involve a significant break with what we take to be the guiding tenets of the dynamic strict analysis of counterfactuals and would risk undercutting the original motivations for the view.

The central motivation for Dynamic SA is that there are essentially dynamic facts concerning how the order of the sentences in a discourse affects its overall coherence. The contrast between Sobel sequences and reverse Sobel sequences nicely illustrates this phenomenon. (5), repeated below, sounds fine.

- (5) a. If I had struck the match, it would have lit.
 b. But of course, if I had struck the match *and* it had been soaked in water last night, it would not have lit.

But when the sequence is uttered in reverse order, it sounds marked, as in (7), repeated below.

- (7) a. If I had struck the match and it had been soaked in water last night, it would not have lit.
 b. But if I had struck the match, it would have lit.

What explains this contrast, according to proponents of Dynamic SA, is that the counterfactual domain easily expands to include new possibilities that we previously ignored, but does not easily contract to exclude possibilities introduced earlier in the discourse, no matter how remote they are.

The problem with the Independence Presupposition is that it will often require us to contract the counterfactual domain, eliminating possibilities introduced earlier in the sequence. But if we are allowed to use the Independence Presupposition to contract the counterfactual domain, it is no longer clear that Dynamic SA can explain why reverse Sobel sequences like (7) are infelicitous. The fact that the match is dry is causally independent of whether I strike the match. This means that we must hold fixed the fact that the match is dry when we evaluate (7-b). Specifically, (7-b) must be evaluated against a counterfactual domain that contains only worlds where the match is dry. But if we evaluate (7-b) against a counterfactual domain containing only worlds where the match is dry, (7-b) will be predicted true. And that is the wrong result.

Although the proponents of Dynamic SA cannot endorse the Independence Presupposition as stated, they can endorse a weaker version of this principle. We call it the Weak Independence Presupposition.

- (19) *Weak Independence Presupposition.* $\llbracket A \square \rightarrow C \rrbracket^{c,w}$ is defined only if where c' is the immediately previous context of utterance, if $h_{\leq c'} \subseteq \llbracket B \rrbracket^{c'}$, then if $\llbracket B \rrbracket^{c,w} = 1$ and B is causally independent of A , $h_{\leq c} \subseteq \llbracket B \rrbracket^c$.

What does Dynamic SA, augmented with the Weak Independence Presupposition, predict about *Dice*? When we evaluate (14-a) and (14-b), we must hold fixed all facts that are independent of Alice's roll and Billy's roll. So, in particular, we must hold fixed the fact that Carol rolls odd. More specifically, (14-a) and (14-b) are defined only if every world in the counterfactual domain where Alice and Billy roll the same type of number is one where Carol still rolls odd. If this condition is met, both (14-a) and (14-b) will come out true. The Weak Independence Presupposition seems to be getting the right results so far.

Now turn to (14-c). To predict a false reading of this sentence, the Weak Independence Presupposition must induce a context change. Specifically, it must ensure that (14-c) is evaluated relative to a counterfactual domain that contains some worlds where Carol rolls even. But the Weak Independence Presupposition will not bring about the needed context change. To predict a false reading of (14-c), the domain must *expand* when we evaluate this sentence. But, as stated, the

Weak Independence Presupposition will never require this sort of change. It tells us which facts we are required to hold fixed, and thus will often force the counterfactual domain to contract. But the Weak Independence Presupposition does not tell us what we must *not* hold fixed, and thus it will never require the domain to *expand*. This means that the Weak Independence Presupposition alone cannot help us to predict a false reading of (14-c).

We need to go beyond causal independence. We want to say that the fact that the antecedent of (14-c) is *not* independent of Carol's roll forces the domain to expand to include worlds where Carol rolls an even number. It is not entirely clear to us how this should be done. But even if an appropriate proposal can be found, we think that it will face a problem similar to the second one faced by Gillies's proposal.

To see this, suppose we have found a way for Dynamic SA to predict that at the end of our original sequence (14-c) sounds false. In other words, we have found a way to predict that the final sentence of the following sequence is true.

- (20) a. If Alice and Billy had thrown the same type of number, then at least one person would still have won \$10.
 b. If Alice and Billy had thrown the same type of number, then Alice, Billy, and Carol could have *all* thrown the same type of number. (Because they could have all thrown odd.)
 c. It is not true that if Alice, Billy, and Carol had all thrown the same type of number, then at least one person would still have won \$10.

If (20-c) is true, it must be evaluated against a counterfactual domain containing worlds where all three of Alice, Billy, and Carol throw an even number. But if that is the case, it should sound fine to continue the sequence as follows:

- (21) So if Alice and Billy had thrown the same type of number, then Carol might have thrown even after all.

Why should (21) sound true, according to Dynamic SA? If (20-c) is true, then there is a world in the counterfactual domain where Carol throws even. But if (21) is evaluated against a domain that contains a world where Carol rolls even, Dynamic SA predicts that (21) is true.

But this looks like the wrong result. Asserting (21) at the end of the sequence makes it sound as if we have changed our minds about whether Carol's roll is independent of Alice's roll and Billy's roll. But of course we have not. (21) does not sound true even after asserting (20-c). Dynamic SA will struggle to predict this if utterances of (14-c)

and (20-c) expand the domain, admitting worlds where Carol rolls even.²¹

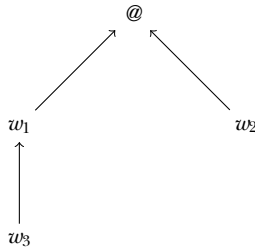
VI. VARIABLY STRICT SEMANTICS

By its very structure, SA is committed to Strengthening with a Possibility. No assumptions about its underlying closeness relation were needed to prove this. Not so for VSA. Strengthening with a Possibility is not written into the semantics of VSA; rather, it corresponds to a certain formal constraint on the closeness ordering $\preceq_{c,w}$, which has been enforced by many of VSA's proponents, including Stalnaker and Lewis.²² We show that by removing that constraint, we can render Strengthening with a Possibility invalid. The resulting logic is that defended by Angelika Kratzer.²³ Finally, we show how to use a Kratzerian *ordering source* to generate an ordering with the right structure in a principled way.

VI.1. *Where We Want to Go.* Here is a model that has the structure we need. Suppose we have four worlds:

- @ is the actual world. (Alice and Carol throw odd; Billy throws even.)
- In w_1 , Alice and Billy throw even; Carol throws odd.
- In w_2 , Alice, Billy, and Carol all throw odd.
- In w_3 , Alice, Billy, and Carol all throw even.

Now suppose we have an ordering that looks like this:



²¹Notice that the Weak Independence Presupposition will not explain why (21) is false. Why not? Although Carol's roll is causally independent of Alice's roll and Billy's roll, the counterfactual domain against which we evaluate (20-c) is one that contains worlds where Carol rolls even. This means that the antecedent of the Weak Independence Presupposition is not satisfied, and so we are not required to hold fixed the fact that Carol rolls odd when we evaluate (21).

²²Both say that whatever else is true about the ordering on worlds, it is *total*: for every w_1 and w_2 either $w_1 \preceq_w w_2$ or $w_2 \preceq_w w_1$. Total orderings rule out incomparabilities that are essential to the model we give below.

²³See Angelika Kratzer, "Partition and Revision: The Semantics of Counterfactuals," *Journal of Philosophical Logic*, x, 2 (May 1981): 201–16.

The arrows represent similarity to @. For example, the arrow going from w_3 to w_1 indicates that w_1 is more similar to @ than w_3 is. (We omit transitive and reflexive arrows.) If there is no arrow between two worlds, then they are *incomparable* with respect to their similarity to @: neither is more similar to @ than the other, and they are not equally similar to @.

In this model, @ $\prec_{@}$ w_1 , @ $\prec_{@}$ w_2 , and $w_1 \prec_{@}$ w_3 . w_1 and w_2 are incomparable, and w_2 and w_3 are incomparable.

The closest worlds to @ where Alice and Billy throw the same type of number are w_1 and w_2 : $f(\text{Alice and Billy same}, @) = \{w_1, w_2\}$. In both worlds, Carol throws odd and wins \$10. Moreover, in one of these worlds—specifically, w_2 —Alice, Billy, and Carol *all* throw the same type of number. This means that (14-a) and (14-b), repeated below, are true.

- (14-a) If Alice and Billy had thrown the same type of number, someone would still have won \$10.
- (14-b) If Alice and Billy had thrown the same type of number, Alice, Billy, and Carol might have all thrown the same type of number.

Finally, the closest worlds to @ where Alice, Billy, and Carol all throw the same are w_2 and w_3 : $f(\text{Alice, Billy, Carol same}, @) = \{w_2, w_3\}$. (Why is w_3 included? Because the only worlds that are strictly closer to @ than w_3 are w_1 and @ itself; but Alice, Billy, and Carol do *not* roll the same in either of these.) In w_2 , Alice, Billy, and Carol throw odd, so they all win. But in w_3 , they throw even, so *nobody* wins. (14-c), repeated below, is false:

- (14-c) If Alice, Billy, and Carol had all thrown the same type of number, someone would still have won \$10.

We have successfully modeled a failure of Strengthening with a Possibility: (14-a) and (14-b) are true, yet (14-c) is false.

The incomparabilities in our model are crucial. Say that the closeness ordering \preceq_w is *almost-connected* just in case $\forall w_1, w_2, w_3 : (w_1 \prec_w w_2 \rightarrow (w_1 \prec_w w_3) \vee (w_3 \prec_w w_2))$. If w_1 is closer to w than w_2 is, then for any third world w_3 , either w_1 is closer to w than w_3 is, or w_3 is closer to w than w_2 is. Simplifying, if w_1 beats w_2 , then either w_1 beats w_3 , or w_3 beats w_2 . Where \preceq_w is a partial order, Strengthening with a Possibility is valid just in case \preceq_w is almost-connected.²⁴ Our model predicts that Strengthening with a Possibility fails precisely because it fails to be almost-connected.

²⁴ *Proof*: \Rightarrow : Our model that follows demonstrates that if Strengthening with a Possibility is valid, then \preceq is almost-connected. If a frame is not almost-connected, then we can build a model on it like the one in the text.

VI.2. *Adding a Kratzerian Ordering Source.* We have given an ordering that allows VSA to predict the right judgments in *Dice*. But how do we ensure that the context *actually supplies* an ordering with this structure?

Our suggestion is to follow Kratzer's influential semantics for modals and conditionals and posit an extra contextual parameter—an *ordering source*, a function g that takes a world w and returns a set of propositions.^{25,26} This set of propositions represents the facts about w that the speakers judge relevant to determining similarity. We then define our ordering in terms of those propositions. w_1 is at least as close to w as w_2 is if and only if it makes true all the same ordering source propositions as w_2 , and possibly more. Formally:

$$(22) \quad w_1 \preceq_w w_2 \text{ iff } \{p \in g(w) : w_1 \in p\} \supseteq \{p \in g(w) : w_2 \in p\}$$

w_1 is at least as close to w as w_2 is just in case every proposition in $g(w)$ that is true in w_2 is also true in w_1 . w_1 is strictly closer to w than w_2 is

⇐: Suppose that \preceq is almost-connected and suppose that, for contradiction, that Strengthening with a Possibility is not valid. Then there is some world w_1 such that $A \Box \rightarrow C$ and $A \Diamond \rightarrow B$ are true there but $A \wedge B \Box \rightarrow C$ is not. This means that $f(A, w_1) \subseteq C$, there is a world $w_2 \in f(A, w_1)$ such that B is true at w_2 and there is a world $w_3 \in f(A \wedge B, w)$ such that $\neg C$ is true there. w_3 cannot be in $f(A, w)$: unlike w_3 , all worlds in $f(A, w)$ are C worlds. By definition of f , this means that there must be some world w_4 in $f(A, w)$ such that $w_4 \prec_{w_1} w_3$.

Now consider whether either $w_4 \prec_{w_1} w_2$ or $w_2 \prec_{w_1} w_3$. In fact, the first disjunct cannot hold: by definition of f , if it did then w_2 would not be in $f(A, w_1)$ after all. But the second disjunct cannot be true either. Again by definition of f , if it did then w_3 would not be in $f(A \wedge B, w_1)$. But now it follows that (contrary to our supposition) that \preceq is not almost-connected: $w_4 \prec_{w_1} w_3$ but neither $w_4 \prec_{w_1} w_2$ nor $w_2 \prec_{w_1} w_3$. So if \preceq is almost-connected, Strengthening with a Possibility must be valid.

To the best of our knowledge, this result was first shown in Frank Veltman, "Logics for Conditionals," PhD diss., University of Amsterdam, 1985.

²⁵If we were to spell out the semantics in full detail, we would also need to add a Kratzerian *modal base* to our semantics, a function from worlds to sets of propositions. For us, the modal base will hold fixed certain key facts about the scenario, namely the rules of the particular game being played. On our way of thinking of things, the modal base represents the background facts which we hold fixed in evaluating counterfactuals, and the ordering source represents the facts we allow to vary and that contribute to closeness.

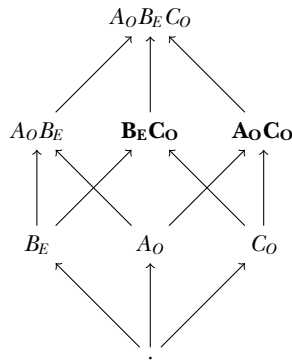
²⁶While we add an ordering source to our semantics, we put it to work in a different way than Kratzer. Kratzer's ordering sources for counterfactuals are *totally realistic*: the intersection of the ordering source propositions is the set containing just the world of evaluation. Her modal bases, on the other hand, are empty. We disagree with Kratzer on both points. We treat modal bases and ordering sources differently in order to predict the failure of Strengthening with a Possibility, but we suspect there are other important advantages as well. Among other things, we appear to avoid the problems Kratzer attempts to solve in Kratzer, "Partition and Revision," *op. cit.*; and Angelika Kratzer, "An Investigation of the Lumps of Thought," *Linguistics and Philosophy*, xii (October 1989): 607–53.

just in case every proposition in $g(w)$ that is true in w_2 is true in w_1 , and some proposition in $g(w)$ that is true in w_1 is false in w_2 .

In our example, the relevant facts are those that concern who got what type of number. We assume, then, that the ordering source looks like this:

$$(23) \quad g(w) = \{Alice \text{ gets odd}, Billy \text{ gets even}, Carol \text{ gets odd}\}$$

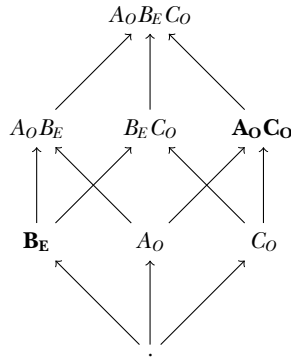
Let A_O , B_E , and C_O be the propositions that Alice rolls odd, Billy rolls even, and Carol rolls odd, respectively. The ordering source in (23) gives rise to the following ordering:



In the top-ranked worlds, things are just as they actually are—Alice and Carol roll odd, and Billy rolls even. Next we have worlds where things differ in *one* respect—worlds where either Alice or Carol rolls even instead of odd, or Billy rolls odd instead of even. Then we have worlds differing in *two* respects, and finally, worlds where *everything* is different—Alice and Carol roll even, and Billy rolls odd.

Let us see how VSA predicts the right judgments in *Dice* using this ordering. The closest worlds where Alice and Billy throw the same type of number are in bold. In both worlds, Carol rolls odd and wins \$10, so (14-a) is true: If Alice and Billy had rolled the same, one person would still have won \$10. Moreover, in one of the closest worlds where Alice and Billy roll the same, Alice, Carol, and Billy all roll odd. So (14-b) is also true: If Alice and Billy had rolled the same, Alice, Billy, and Carol might have all rolled the same.

Finally, turn to (14-c), which says that if Alice, Billy, and Carol had rolled the same, someone would still have won \$10. We find the worlds where Alice, Billy, and Carol roll the same type of number. They are now in bold below:



We have worlds where Alice, Billy, and Carol all throw odd (top right) and worlds where they all throw even (bottom left). These are incomparable—neither is closer to actuality than the other is. (The reason they are incomparable is that the sets of ordering source propositions true at each are disjoint.) Both are among the closest worlds where Alice, Billy, and Carol roll the same. So (14-c) is false: In some of the closest worlds where they throw the same, they throw even, and nobody wins.²⁷

VI.3. Why Does Strengthening with a Possibility Seem Valid? We argued that Strengthening with a Possibility has counterexamples, and we offered a variably strict account that invalidates it. Still, the inference often *seems* valid. Suppose I confidently say that if I had taken modal logic last semester, I would have passed. You reply that if I had taken the course, it might have been taught by Joe, who is notorious for his difficult problem sets and harsh grading. If I accept your response, I seem to have two options. I could stand firm, insisting that I would have passed even Joe's challenging course, or I could retreat, rescinding my earlier claim that I would have passed the class. What I *cannot* do is maintain that I would have passed the course, even though I might not have passed a course taught by Joe. That I do not have this option is only explained if Strengthening with a Possibility does not fail in this particular case. We must place certain constraints on when the inference can fail. We want it to fail in *Dice*, but not here.

We give a pragmatic explanation of this difference. Our idea is that Strengthening with a Possibility fails only when there is more than one relevant way of making the antecedent true. What are the rele-

²⁷We are not committed to using an ordering source that simply records who gets what type of number in every version of this case. It seems to be the natural one here, but a case with a more complicated structure may call for a more complicated ordering source.

vant ways of making the antecedent true? We will answer this question by linking ordering sources to salient questions in context and prove that, in the resulting technical sense, Strengthening with a Possibility does indeed only fail when there is more than one way of making the antecedent true.

First, a word about questions. It is standard to let the semantic value of a question be the set of its *answers*. Following Groenendijk and Stokhof, we will assume the set of answers is a set of propositions that are mutually exclusive and exhaustive.²⁸ That is, questions are a *partition* of the set of worlds. So, for instance, the value of the question *Which of Alice and Bob ate cookies?* would be the set containing the propositions *Alice and Bob ate cookies*, *Alice but not Bob ate cookies*, *Bob but not Alice ate cookies*, and *Neither Alice nor Bob ate cookies*. Some questions make more distinctions than others. For instance, the question *Which of Alice and Bob ate cookies?* does not distinguish between worlds where *Carol* ate cookies and worlds where she did not; worlds of both kinds will be contained in every answer to this question. The question *Which of Alice, Bob, and Carol ate cookies?* does make such distinctions: all the worlds in any given cell agree on whether *Carol* ate cookies. In other words, the question *Which of Alice, Bob, and Carol ate cookies?* divides up the set of worlds more finely than the question *Which of Alice and Bob ate cookies?* does. Representing questions with partitions captures this: every cell in the value of *Which of Alice, Bob, and Carol ate cookies?* is a subset of some cell in the value of *Which of Alice and Bob ate cookies?*, but not vice versa. The former makes all the distinctions of the latter, and more besides. As is standard, we will say that *Q'* refines *Q* if and only if every element of *Q'* is a subset of some element of *Q*.

Because questions can distinguish more or less finely between worlds, they have proven to be useful in representing issues that are live in the context. We assume that in contexts in which we utter counterfactuals, certain *non-counterfactual* questions are relevant. So, for instance, in the Joe case above, a question like *Did I study and did Joe teach?* is relevant. These questions plausibly represent the kinds of distinctions we are interested in making. We will assume that there is a most refined such question that captures exactly the level of detail that is relevant in a context. It is to these questions that ordering sources will be linked.²⁹

²⁸ See Jeroen Groenendijk and Martin Stokhof, "Studies on the Semantics of Questions and the Pragmatics of Answers," PhD diss., University of Amsterdam, 1984. Note that we do not really mean to take a stand on the semantics of questions in natural language. We simply use this formalism to represent a way that an issue can be live.

²⁹ Note that these questions must be distinguished from Craige Roberts's notion of a *question under discussion*; see Craige Roberts, "Information Structure in Discourse:

To do this, we first define from the ordering source a function g^- . For any given world, $g^-(w)$ collects together the negations of the propositions in $g(w)$. More formally, $g^-(w) = \{p : \neg p \in g(w)\}$. From g together with g^- we can build a partition on worlds. Construct the set of maximal consistent propositions built out of $g(w) \cup g^-(w)$; call it G_w . Since G_w is a partition, it represents a question that we might be concerned with in our context.

We connect this question, G_w , to the relevant questions in context. Say that Q_c is the most refined salient (non-counterfactual) question in c . We propose that the partition we can construct from our ordering source, G_{w_c} , just is the most refined salient question Q_c . This has the effect that the ordering source cannot distinguish between worlds in ways that are not already present in the most refined salient question. This does not pin g down uniquely, but does impose a substantial constraint on it: whatever g_c is, it must be the case that $G_{w_c} = Q_c$. We call this constraint *Question Sensitivity*.

Given *Question Sensitivity*, we can prove that we get failures of Strengthening with a Possibility only if there are two distinct answers to Q_c that realize the antecedent of the final strengthened conditional.

There are A, B, C such that $\llbracket A \Box \rightarrow C \rrbracket^{c,w,g} = 1$, $\llbracket A \Diamond \rightarrow B \rrbracket^{c,w,g} = 1$, and $\llbracket A \wedge B \Box \rightarrow C \rrbracket^{c,w,g} = 0$ only if $\exists p, q \in Q_c : p \models A \wedge B$ and $q \models A \wedge B$ and $p \neq q$.³⁰

Towards an Integrated Formal Theory of Pragmatics," in Jae Hak Yoon and Andreas Kathol, eds., *Ohio State University Working Papers in Linguistics, Volume 49* (Columbus: Ohio State University, 1996), pp. 91–136. Among other things, the current question under discussion must be *unanswered* in the context, a feature our questions do not share. (If this is felt to be unintuitive, our partitions can instead, following David Lewis, be thought of as relevant subject matters. See David Lewis, "Statements Partly about Observation," *Philosophical Papers*, xvii, 1 (1988): 1–31.)

³⁰ *Proof.* Suppose that $A \Box \rightarrow C$, $A \Diamond \rightarrow B$, but $A \wedge B \Box \rightarrow C$ is false. For contradiction, suppose there is just one cell that makes $A \wedge B$ true. Call it Q . We appeal to three facts:

1. All worlds in a partition cell are equally good.
(This is because they all make the same ordering source propositions true.)
2. $Q = Q \cap A = Q \cap (A \wedge B)$.
(This is because Q already contains only $A \wedge B$ worlds.)
3. $Q \cap (A \wedge B) = f(A \wedge B, w)$
(This follows from the definition of f plus the fact that Q is the only $A \wedge B$ cell.)

Either $Q \cap A \subseteq f(A, w)$ or it is not. Suppose it is. Then, by facts 2 and 3, $f(A \wedge B, w) \subseteq f(A, w)$. And since $f(A, w) \subseteq C$, $A \wedge B \Box \rightarrow C$ is true, contrary to our supposition. Suppose it is not. Then $f(A, w)$ contains *no* B -worlds: Q contains the only $A \wedge B$ worlds and, by fact 1, they are all equally good. So $A \Diamond \rightarrow B$ is false, contrary to our supposition.

Put informally, the reason for this is as follows: if there is only one partition cell, call it p , that entails $A \wedge B$, then either p is a subset of the closest A -worlds or not. If it is, then if $A \Box \rightarrow C$ is true, all the closest $A \wedge B$ -worlds will have to be C -worlds. If it is not, then since all worlds in p are equally close, no B -worlds will be among the closest A -worlds and so $A \Diamond \rightarrow B$ will be false.

To see how this helps, let us return to the case of Joe. Here quite plausibly the most salient question is *Did I take logic and did Joe teach?*, which gives us the following partition:

{*I take logic and Joe teaches, I take logic and Joe does not teach,*
I do not take logic and Joe teaches, I do not take logic and Joe does not teach}

There is only one cell of the partition which makes true our strengthened antecedent, namely *I take logic and Joe teaches*. We showed that, when there is only one cell entailing the strengthened antecedent, Strengthening with a Possibility cannot fail. So we rightly predict it should seem good here.

We also allow for it to fail in *Dice*. There G_{w_c} is the following partition:

Only Alice gets odd, Only Billy gets odd, Only Carol gets odd, Only Alice and Billy get odd, Only Billy and Carol get odd, Only Alice and Carol get odd, No one gets odd, Everyone gets odd

Here we do have more than one way of making the strengthened antecedent true: *No one gets odd* and *Everyone gets odd*. So our constraint predicts that Strengthening with a Possibility can fail here, just as we want.

This is an interesting, and previously untapped, advantage of using an ordering source. Not only can it offer an account of when Strengthening with a Possibility fails, but it can also be used to explain why the inference often seems to go through. In cases like ours where Strengthening with a Possibility fails, we are interested in *different* ways in which the antecedent could be true. In simple cases, we do not make such fine distinctions and so the inference seems valid.

VII. DISJUNCTIVE ANTECEDENTS

The last order of business is to address a potential concern about our case. We say that (14-c), repeated below, is false in *Dice*:

(14-c) If Alice, Billy, and Carol had all rolled the same type of number, someone would still have won \$10.

Why do we judge (14-c) false? A natural explanation is that there are two ways for Alice, Billy, and Carol to roll the same type of number—

by all rolling odd and by all rolling even. If they had all rolled odd, someone—indeed, all of them—would have won \$10. But not if they all rolled even. If they had all rolled even, nobody would have won. We reject (14-c) because one way of rolling the same is not a way of winning \$10.

Many will worry about this reasoning. It seems to rely on an inference pattern that is perilously similar to a controversial inference pattern known as *Simplification of Disjunctive Antecedents* (hereafter “Simplification”). It is far from obvious that Simplification is valid. And so if the judgment that (14-c) is false does indeed rely on Simplification, that is reason enough to question it. We will argue that despite appearances, we are not relying on Simplification but a close relative of that principle, which we call *Weak Simplification with a Possibility*. And we will show that, unlike Simplification *itself*, Weak Simplification with a Possibility is validated by VSA.

First let us articulate the worry in more detail. Simplification says that from a counterfactual $(A \vee B) \Box \rightarrow C$ one can infer $A \Box \rightarrow C$ and $B \Box \rightarrow C$.³¹ Formally:

$$(24) \text{ Simplification. } A \vee B \Box \rightarrow C \models A \Box \rightarrow C \wedge B \Box \rightarrow C$$

Let us look at an example. Consider (25):

- (25) If I had taken the bus or the subway to work, I would have arrived at work on time.

(25) strongly suggests that both (26) and (27) are true:

- (26) If I had taken the bus to work, I would have arrived on time.
 (27) If I had taken the subway to work, I would have arrived on time.

Why think that our judgment that (14-c) is false relies on something like Simplification? The idea behind Simplification does not essentially involve disjunction. There are two salient ways of making the antecedent true. I could take the bus to work, or I could take the subway to work. We can think of Simplification as saying that the counterfactual is true only if *every salient way of making the antecedent true* is also a way of making the consequent true. This explains why we judge (25) false when, say, (26) is false. To say that (26) is false is to say that one salient way of making the antecedent true is not a way of making the consequent true.

³¹The appeal of Simplification was noted in Kit Fine, “Critical Notice of Lewis’s *Counterfactuals*,” *Mind*, LXXXIV, 451 (July 1975): 451–58; and, independently, in Donald Nute, “Counterfactuals and the Similarity of Worlds,” this JOURNAL, LXXII, 21 (December 1975): 773–78.

But *that* reasoning looks very much like the reasoning that lead us to reject (14-c). We said that we judge (14-c) false because there are two ways for Alice, Billy, and Carol to roll the same type of number—they could all roll odd or they could all roll even—and only *one* of them is a way of winning \$10.

Why should we be concerned about all of this? Because VSA invalidates Simplification. To see this, suppose that in all of the closest worlds where I take the bus, I arrive on time, but in some of the closest worlds where I take the subway, I arrive late. Moreover, the worlds where I take the bus are closer than those where I take the subway. In this scenario, (25) is true. In all of the closest worlds where I take the subway or the bus, I take the bus, so arrive on time. But (27) is false. In some of the closest worlds where I take the subway, I arrive late.

So Simplification is not valid according to VSA. But, as we said, our judgment that (14-c) is false seems to rely on something like Simplification. But surely we should not rely on an inference pattern that our own theory invalidates.

There is, however, reason to doubt that Simplification really is the principle we rely on. It is well known that the Simplification inference is sometimes *canceled*. Suppose you are asked which side Spain fought for in World War II, and you reply: “Spain did not fight in the war. But if she had joined either the Axis or the Allies, she would have joined the Axis.”³² No reasonable interlocutor would object on the grounds that if Spain had joined the Axis, she would not have joined the Allies.

But (14-c) is structurally similar to the Spain counterfactual. The antecedent, *Alice, Billy, and Carol roll the same type of number*, is equivalent to the disjunction, *Alice, Billy, and Carol roll odd or Alice, Billy, and Carol roll even*. Given knowledge of the game, the consequent *someone wins* is equivalent to *someone rolls odd*. Together with what is given in the antecedent, *someone rolls odd* is equivalent to the proposition, *Alice, Billy, and Carol all rolled odd*. This means that (14-c) is contextually equivalent to:³³

- (28) If Alice, Billy, and Carol had all rolled odd or all rolled even, they would have all rolled odd.

³²This example is due to Thomas McKay and Peter van Inwagen, “Counterfactuals with Disjunctive Antecedents,” *Philosophical Studies*, xxxi, 5 (May 1977): 353–56.

³³We rely on the following notion of contextual equivalence. Where C is a Stalnakerian context (that is, the set of worlds representing the information compatible with what is known by the conversational participants), $\llbracket A \rrbracket^c$ and $\llbracket B \rrbracket^c$ are contextually equivalent in C just in case, for all $w \in C$: $\llbracket A \rrbracket^{c,w} = \llbracket B \rrbracket^{c,w}$. Informally, A and B are contextually equivalent just in case they are equivalent given what is known in the context.

But (28) is just like the Spain counterfactual: ‘If Spain had joined the Axis or the Allies, she would have joined the Axis’. If we are not tempted by Simplification in the Spain case, we should not be tempted by it in our case, either.³⁴

There is another reason to doubt that we rely on Simplification. As Alonso-Ovalle notes, Simplification inferences are also tempting for *might*-counterfactuals.³⁵ Take (29):

(29) If I had taken the bus or the subway, I might have arrived on time.

(29) strongly suggests that both (30) and (31) are true:

(30) If I had taken the bus, I might have arrived on time.

(31) If I had taken the subway, I might have arrived on time.

But Simplification is sometimes canceled with *might*-counterfactuals, just as it is with *would*-counterfactuals. Take (32), for example:

(32) If I had taken the bus or the subway, I might have taken the subway.

When you hear (32), we take it you are not inclined to infer that if I had taken the bus, I might have taken the subway. Simplification is canceled in (32).

Return now to *Dice*, and consider the following *might*-counterfactual:

(33) If Alice, Billy, and Carol had all rolled the same, someone might have won \$10.

(33) seems true. If Alice, Billy, and Carol had all rolled the same, they might have all rolled odd, and so they might have all won. This is another example in which the Simplification inference is canceled. For if it were not, the sentence would sound absurd. It is plainly not the case that if they had all rolled *even*, someone might have won \$10. Since (33) sounds true, Simplification is not in effect.

³⁴This is precisely why dynamic SA predicts that (14-c) is true, even though it (Strawson-)validates Simplification. The domain at this point will contain only worlds where they all get odd. This means the counterfactual

(i) If Alice, Billy, and Carol had all rolled even, someone would have won \$10

is *undefined*; and in cases where the conclusion is undefined, a Strawson-valid inference will not seem compelling.

³⁵See Luis Alonso-Ovalle, “Counterfactuals, Correlatives, and Disjunction,” *Linguistics and Philosophy*, xxxii, 2 (July 2009): 207–44.

(33) is the *might*-counterfactual analogue of (14-c). (14-c) says that if Alice, Billy, and Carol had all rolled the same type of number, someone would still have won \$10; (33) says that if Alice, Billy, and Carol had all rolled the same type of number, someone *might* have won \$10. If Simplification were in effect for (14-c), we would also expect it to be in effect for (33). But it is not. Since (33) sounds true, Simplification must be canceled.

It is unlikely, then, that our intuitions about (14-c) rely on Simplification. But we think we are relying on a close relative of that principle. We call the principle *Weak Simplification with a Possibility*.³⁶ We divide the principle into two inference patterns:

$$(34) \quad (A \vee B) \Box \rightarrow C, (A \vee B) \Diamond \rightarrow A \vDash A \Diamond \rightarrow C$$

$$(35) \quad (A \vee B) \Box \rightarrow C, (A \vee B) \Diamond \rightarrow B \vDash B \Diamond \rightarrow C$$

³⁶You might be surprised that we do not rely on a more familiar inference pattern, which we will call Simplification with a Possibility:

$$(i) \quad (A \vee B) \Box \rightarrow C, (A \vee B) \Diamond \rightarrow B, (A \vee B) \Diamond \rightarrow A \vDash A \Box \rightarrow C \wedge B \Box \rightarrow C$$

This inference pattern has been used in attempts to derive Simplification as an implicature. For instance, Jonathan Bennett claims that disjunctive antecedent counterfactuals implicate both of the *might*-counterfactuals, and so what look like instances of Simplification are in fact instances of Simplification with a Possibility; see Jonathan Bennett, *A Philosophical Guide to Conditionals* (Oxford: Oxford University Press, 2003).

The reason we do not rely on Simplification with a Possibility is simple: it is equivalent to Strengthening with a Possibility, given standard classical assumptions about counterfactuals.

To show this, we will assume Substitution of Logical Equivalents holds in the antecedent of counterfactuals; and the further principle that $A \Diamond \rightarrow B$ entails $A \Diamond \rightarrow A \wedge B$, which we will call Right Monotonicity. (Both of these are validated by standard theories of counterfactuals.)

Simplification with a Possibility \Rightarrow Strengthening with a Possibility:

- | | |
|--|---|
| 1. $A \Box \rightarrow C$ | <i>Assumption</i> |
| 2. $A \Diamond \rightarrow B$ | <i>Assumption</i> |
| 3. $(A \wedge B) \vee (A \wedge \neg B) \Box \rightarrow C$ | 1, <i>Substitution of Logical Equivalents</i> |
| 4. $A \Diamond \rightarrow (A \wedge B)$ | 2, <i>Right Monotonicity</i> |
| 5. $(A \wedge B) \vee (A \wedge \neg B) \Diamond \rightarrow (A \wedge B)$ | 4, <i>Substitution of Logical Equivalents</i> |
| 6. $(A \wedge B) \Box \rightarrow C$ | 3,5, <i>Simplification with a Possibility</i> |

Strengthening with a Possibility \Rightarrow Simplification with a Possibility:

- | | |
|---|---|
| 1. $(A \vee B) \Box \rightarrow C$ | <i>Assumption</i> |
| 2. $(A \vee B) \Diamond \rightarrow A$ | <i>Assumption</i> |
| 3. $(A \vee B) \wedge A \Box \rightarrow C$ | 1,2, <i>Strengthening with a Possibility</i> |
| 4. $A \Box \rightarrow C$ | 3, <i>Substitution of Logical Equivalents</i> |

And unlike Simplification itself, this close relative is validated by VSA.³⁷

There are two differences between Weak Simplification with a Possibility and Simplification. First, Weak Simplification with a Possibility yields weaker conclusions—that C might have occurred, had A occurred (in the case of (34)), and that C might have occurred, had B occurred (in the case of (35)). Second, Weak Simplification with a Possibility requires more assumptions. To infer $A \diamondrightarrow C$ from $(A \vee B) \squarerightarrow C$, we must know that A might have been true, if $(A \vee B)$ had been true. Similarly, to infer $B \diamondrightarrow C$ from $(A \vee B) \squarerightarrow C$, we must know that B might have been true, had $(A \vee B)$ been true.

Unlike Simplification itself, (34) and (35) are validated by VSA. To see this, take (34). If the *might*-counterfactual premise is true, then some of the closest $(A \vee B)$ -worlds are A -worlds. Furthermore, those A -worlds must constitute at least some of the closest A -worlds. So, if $(A \vee B) \squarerightarrow C$ is true, some of the closest A -worlds are C -worlds.

Let us apply this to *Dice*. Recall (14-c), repeated below:

- (14-c) If Alice, Billy, and Carol had all rolled the same type of number, someone would still have won \$10.

Consider the closest worlds where Alice, Billy, and Carol all roll the same. Some of these worlds are worlds where Alice, Billy, and Carol all roll odd, since (36) is true:

- (36) If they had all thrown the same number, they might have rolled even.

By Weak Simplification with a Possibility, (14-c) and (36) entail (37):

- (37) If Alice, Billy, and Carol had all rolled even, someone might have won \$10.

But (37) is clearly false. By the setup of the case, if they had all rolled even, nobody would have won \$10. But if (37) is false and (36) is true, then, by Weak Simplification of a Possibility, it follows that (14-c) is false.

³⁷ *Proof:* Suppose $(A \vee B) \squarerightarrow C$ and $(A \vee B) \diamondrightarrow B$ are true at w_1 . Then $f(A \vee B, w_1) \subseteq C$ and $f(A \vee B, w_1) \cap A \neq \emptyset$.

We first show $f(A \vee B, w_1) \cap A \subseteq f(A, w_1)$. Take an arbitrary $w_2 \in f(A \vee B, w_1) \cap A$. Suppose it were not in $f(A, w_1)$. Then there would have to be a $w_3 \in f(A, w_1)$ such that $w_3 <_{w_1} w_2$. But if $w_3 \in A$ then $w_3 \in A \vee B$. So w_2 would not be in $f(A \vee B, w_1) \cap A$ as $w_3 \in A \vee B$ and $w_3 <_{w_1} w_2$. So $w_2 \in f(A, w_1)$ after all.

But if $f(A \vee B, w_1) \cap A \subseteq f(A, w_1)$, then since $f(A \vee B, w_1) \cap A \subseteq C$, $f(A, w_1) \cap C \neq \emptyset$. So $A \diamondrightarrow C$ is true at w_1 . If we suppose further that $A \vee B \diamondrightarrow B$ is true at w_1 , we can prove in a similar manner that $B \diamondrightarrow C$ is true at w_1 .

We have seen that if Weak Simplification with a Possibility is valid, (14c) must be false. We can also see how Weak Simplification with a Possibility helps make sense of our original judgments about the case. We said that (14c) seems false because it is too committal—it seems to ignore the possibility that Alice, Billy, and Carol roll the same by all rolling even. Weak Simplification with a Possibility explains why (14c) seems committal in just this sense. We know, by the setup of the case, that (37) is false. So if (14c) is to be true, then (36) must be false. But, intuitively, (36) is *true*—if Alice, Billy, and Carol had all rolled the same type of number, it could have easily been that they all rolled even.

The Simplification objection gets *something* right: our intuitions about (14c) rely on something *like* Simplification. But it is not Simplification itself. We claim that we rely on a close relative of that principle, Weak Simplification with a Possibility, and that this principle is validated by VSA.

VIII. CONCLUSION

We suggested that the debate between SA and VSA could be clarified by looking at a wider range of strengthening principles. This suggestion has been borne out. Dynamic SA validates Strengthening with a Possibility. But this inference is not valid. Counterexamples to Strengthening with a Possibility pose a much more serious problem for Dynamic SA than counterexamples to Antecedent Strengthening itself. While Antecedent Strengthening is merely Strawson-valid, Strengthening with a Possibility is *classically* valid. Counterexamples to it do not involve presupposition failure, so the dynamic principles that drive context change do not apply. But if that is right, Dynamic SA has no way to account for counterexamples to Strengthening with a Possibility. VSA, on the other hand, can easily model failures of Strengthening with a Possibility. We conclude that the failure of Strengthening with a Possibility tells strongly against Dynamic SA and in favor of an ordering source-based version of VSA.

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