The Qualitative Thesis*

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1 Introduction

This paper is about

> **The Qualitative Thesis**. If you are not sure that $[\![\neg\varphi]\!]$, then you are sure of the indicative conditional $[\![\text{if } \varphi, \text{then } \psi]\!]$ just in case you are sure of $[\![\psi]\!]$ conditional on $[\![\varphi]\!]$.

Although The Qualitative Thesis has received far less attention than Stalnaker's Thesis, its quantitative cousin, it occupies a central place in contemporary theories of indicative conditionals. It's needed to explain why the indicative conditional often behaves like the material conditional. And it's needed to account for the semantic differences between indicatives and subjunctives.[1]

We investigate the epistemological consequences of The Qualitative Thesis. We characterize The Qualitative Thesis in standard formal frameworks for studying the logic of attitudes and conditionals. With these characterization results in hand, we develop a connection first observed by Ben Holguín (p.c.) between The Qualitative Thesis and a plausible margin for error requirement on rational sureness. Building on [Holguín, 2019], we show that The Qualitative Thesis is inconsistent with the margin for error principle. In response, we propose a new *shifty* semantics for indicative conditionals. We say that the meaning of an indicative conditional is partly determined by the conditional's local informational environment—the conditional's *local context*—which, in turn, is systematically shifted by attitude operators. Our account validates The Qualitative Thesis, but dispenses with its undesirable epistemological consequences.

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[1] We are not the first to discuss a qualitative version of Stalnaker's Thesis. See [Harper, 1975], [Harper, 1976] and [Gärdenfors, 1981]. Our formulation of The Qualitative Thesis is qualified: we restrict attention to the case where the antecedent of the conditional is not ruled out. We say why we focus on this qualified principle in §3; see footnote 10.

We also explore a central commitment of our shifty theory, *Conditional Locality*, and argue that it is independently motivated. It's well known that indicative conditionals often behave like material conditionals. But it has not been appreciated that this phenomenon persists in embedded environments, such as third-personal attitude reports. The full range of data supports what we call *The Strong Qualitative Thesis*. It is a virtue of the shifty account that it validates The Strong Qualitative Thesis and a shortcoming of the non-shifty account that it does not.

The rest of the paper is organized into eight sections. §2 presents a variably strict semantics for indicatives and a restricted version of Stalnaker's Thesis. §3 presents The Qualitative Thesis and explains why it's important. §4 introduces a framework for studying the logic of attitudes and conditionals, characterizes The Qualitative Thesis on a class of frames, and shows that The Qualitative Thesis entails a principle we call *No Opposite Materials*. §5 shows that No Opposite Materials is inconsistent with the margin for error principle; §6 explains the tension in more intuitive terms. §7 presents our account and shows that it validates The Qualitative Thesis while remaining consistent with the margin-for-error principle. §8 defends Conditional Locality and The Strong Qualitative Thesis. §9 concludes.

2 Indicative Conditionals and Stalnaker's Thesis

There are two leading theories of indicative conditionals: the *strict analysis* and the *variably strict analysis*.[2] We concentrate on the variably strict analysis, but as we show in Appendix A.2, our main arguments can be recast in the strict conditional framework. On variably strict theories, $\ulcorner$if φ, then $\psi\urcorner$ is true just in case $[\![\psi]\!]$ is true in all relevant $[\![\varphi]\!]$-worlds. To state this precisely, we posit a contextually-supplied *selection function* f, which takes a world w, and a proposition $[\![\varphi]\!]$, and returns a set of worlds where $[\![\varphi]\!]$ is true. Then:

Variably Strict Semantics. $[\![\text{if } \varphi, \text{ then } \psi]\!]^{c,w} = 1$ iff: $\forall w' \in f_c(w, [\![\varphi]\!]^c) : [\![\psi]\!]^{c,w'} = 1$.

We refer to the value of the selection function $f(w, [\![\varphi]\!]^c)$, for some world w and proposition $[\![\varphi]\!]^c$, as the *selected* $[\![\varphi]\!]^c$*-worlds at* w. With this terminology, the Variably Strict Semantics says that $\ulcorner$if φ, then $\psi\urcorner$ is true at $\langle c, w\rangle$ just in case all the selected $[\![\varphi]\!]^c$-worlds at w are $[\![\psi]\!]^c$-worlds.

Much research about indicative conditionals concerns the probabilities we assign to them. Normally, the probability we assign to $\ulcorner$if φ, then $\psi\urcorner$ is equal to the probability we assign to ψ conditional on φ. Indeed, the empirical support for this equation is so strong that many have been tempted by following thesis:[3]

[2]For defenses of the strict analysis, see [Gillies, 2004], [Gillies, 2009], [Rothschild, 2013], and [Willer, 2017]. For classic defenses of variably strict views, see [Kratzer, 2012] and [Stalnaker, 1975a].

[3]See [Stalnaker, 1970], [Adams, 1975], [van Fraassen, 1976] and [Jeffrey and Edgington, 1991] for theories that are

Stalnaker's Thesis. The probability of $\ulcorner$if φ, then $\psi\urcorner$ is equal to the probability of $[\![\psi]\!]$ conditional on $[\![\varphi]\!]$.

Stalnaker's Thesis is stated in terms of the probabilities of sentences, not propositions, as is usually done. Following [Bacon, 2015] and others, we read this thesis as saying that, where $[\![\varphi]\!]^c$, $[\![\psi]\!]^c$, and $[\![\text{if } \varphi, \text{ then } \psi]\!]^c$ are the propositions expressed by φ, ψ, and $\ulcorner$if φ, then $\psi\urcorner$ in a context c, then for any probability function P, $P([\![\text{if } \varphi, \text{ then } \psi]\!]^c) = P([\![\psi]\!]^c | [\![\varphi]\!]^c)$.

The last few decades have seen a battery of triviality results showing that Stalnaker's Thesis is untenable.[4] But some philosophers have observed that *contextualists* about indicatives can accept a restricted version of Stalnaker's Thesis.[5] To see how contextualism helps, we should say something about the form of contextualism at issue. Indicative conditionals are said to be *information sensitive*. They talk about what's true in antecedent worlds compatible with a relevant body of evidence. Contextualists say the relevant body of evidence is supplieed by the context of utterance. An utterance of $\ulcorner$if φ, then $\psi\urcorner$ in a context c says that $[\![\psi]\!]^c$ is true throughout some set of $[\![\varphi]\!]^c$-worlds compatible with the information possessed by the speakers in c.[6]

The contextually-supplied evidence has two jobs. It partly determines the proposition expressed by an utterance of $\ulcorner$if φ, then $\psi\urcorner$. And it determines the probability function that speakers use to evaluate the probabilities of conditionals. The limited version of Stalnaker's Thesis available to contextualists says that the equation holds when the evidence determining the probability function and the evidence determining the propositional content of the conditional are identical. Where $P_{c,w}$ is the probability function of the speakers in c in world w, this gives us the following 'local' thesis.

The Local Thesis. For any world w and any context c, if $P_{c,w}([\![\varphi]\!]^c) > 0$, then $P_{c,w}([\![\text{if } \varphi, \text{ then } \psi]\!]^c) = P_{c,w}([\![\psi]\!]^c | [\![\varphi]\!]^c)$

The Local Thesis is weaker than Stalnaker's Thesis. It does not say *every* probability function assigns to $[\![\text{if } \varphi, \text{ then } \psi]\!]^c$ the same probability as it assigns to $[\![\psi]\!]^c$ given $[\![\varphi]\!]^c$; it says the probability function *determined by the evidence in* c assigns to $[\![\varphi > \psi]\!]^c$ the same probability as it assigns to $[\![\psi]\!]^c$ conditional on $[\![\varphi]\!]^c$.[7]

designed to predict Stalnaker's Thesis. See [Evans and Over, 2004] and [Douven and Verbrugge, 2013] for empirical work supporting Stalnaker's Thesis.

[4]See [Lewis, 1976] for the first triviality result. There have been many others since. Some of the main ones include [Stalnaker, 1976], [Hájek, 1989], [Hájek and Hall, 1994], and [Edgington, 1996].

[5]See [Bacon, 2015] and [Khoo, 2019] for versions of this strategy and [Rothschild, 2013] for a related approach.

[6]For simplicity, we assume that the contextually-supplied evidence is that of the speaker, but this assumption is not essential to our main argument. What we say here is compatible with other forms of contextualism and with relativism about the relevant information.

[7]See [Bacon, 2015] and [Khoo, 2019] for further discussion. Note that The Local Thesis evades what [Bacon, 2015] calls *dynamic triviality results*, which show that the conditional probability equation cannot hold for *all* probability functions and *all* conditional propositions. But it does not automatically avoid what Bacon terms *static triviality results*,

3 The Qualitative Thesis

The Local Thesis concerns the probabilities of conditionals. We are interested in a *qualitative* analogue of The Local Thesis: *The Qualitative Thesis*.

Let us introduce some operators to talk about what the speaker in a given context is sure of. Let $S^{c,w}([\![\psi]\!]^c)$ mean that the speakers in c is *sure that* $[\![\psi]\!]^c$ in w; and let $S^{c,w}_{[\![\varphi]\!]^c}([\![\psi]\!]^c)$ mean that the speaker in c is sure of $[\![\psi]\!]^c$ conditional on $[\![\varphi]\!]^c$ in w. Then:

> **The Qualitative Thesis**. For any world w and context c, if $\neg S^{c,w}([\![\neg\varphi]\!]^c)$, then: $S^{c,w}([\![$if φ, then $\psi]\!]^c)$ if and only if $S^{c,w}_{[\![\varphi]\!]^c}([\![\psi]\!]^c)$

The Qualitative Thesis says that if the speaker in c is sure of $[\![\neg\varphi]\!]^c$, then she is sure of $[\![$if φ, then $\psi]\!]^c$ just in case she is sure of $[\![\psi]\!]^c$ conditional on $[\![\varphi]\!]^c$. This thesis, and its epistemological consequences, will be the focus of this paper.

Two preliminary remarks are in order.

First, a word about conditional sureness. Although one can in principle make sense of the notion of sureness conditional on $[\![\varphi]\!]$ even when one is sure that $[\![\varphi]\!]$ is false, we will set this case aside; we assume $[\![\varphi]\!]$ is compatible with what the subject is sure of. We also make the following assumption.

> **Rational Monotonicity**. If you are sure that $[\![\psi]\!]$ and you are not sure that $[\![\neg\varphi]\!]$, then you are sure of $[\![\psi]\!]$ conditional on $[\![\varphi]\!]$.

Rational Monotonicity is a standard assumption throughout the literature on belief revision.[8] In a probabilistic framework, it is a consequence of the Ratio Formula analysis of conditional probability. And it follows from the axioms of AGM, the dominant theory of qualitative belief revision. With Rational Monotonicity, conditional sureness can be understood in more familiar terms. Specifically, to say that you are sure of $[\![\psi]\!]$ conditional on $[\![\varphi]\!]$ is just to say that you are

such as those due to [Stalnaker, 1976] and [Fitelson, 2015], which purport to show that *no* probability function assigns to $[\![$if φ, then $\psi]\!]^c$ the probability that it assigns to $[\![\psi]\!]^c$ conditional on $[\![\varphi]\!]^c$. Static triviality results tend to rely on more assumptions about indicative conditionals. One option for proponents of The Local Thesis is to reject these principles. [Bacon, 2015] pursues this strategy in response to Stalnaker's static triviality result. A different response is to limit the scope of The Local Thesis. For example, one might take a cue from [McGee, 1989] and restrict The Local Thesis to conditionals with non-conditional antecedents; similarly, in response to Fitelson's triviality result, one might follow [Khoo and Mandelkern, 2019] and restrict The Local Thesis to conditionals with non-conditional consequents. We will not take a stand on how best to respond to static triviality results.

[8] [Gärdenfors, 1986] proves a triviality result for The Qualitative Thesis that takes Rational Monotoncity as a premise. However, by assuming the Qualitative Thesis holds for a given interpretation of the conditional and any arbitrary belief state, Gärdenfors effectively assumes a global version of the Qualitative Thesis

> **Global Qualitative Thesis.** For any world w and any contexts c and c': $S^{c,w}([\![$if φ, then $\psi]\!]^{c'})$ if and only if $S^{c,w}_{[\![\varphi]\!]^{c'}}([\![\psi]\!]^{c'})$

This thesis is much stronger than The Qualitative Thesis and for the contextualist should be as undesirable as (non-local) Stalnaker's Thesis.

sure of the material conditional $[\![\neg\varphi \text{ or } \psi]\!]$. (Note that we temporarily drop the assumption of Rational Monotonicity in §4 in order to give a general characterization of The Qualitative Thesis, but for the rest of the paper we take it for granted.)

Second, note that The Qualitative Thesis is restricted in just the way that The Local Thesis is. It doesn't say that just *anyone* must be sure of $[\![\text{if } \varphi, \text{ then } \psi]\!]^c$ just in case they are sure of ψ conditional φ. It says that *if your total evidence is identical to the evidence of the speakers in c*, then you are sure of $[\![\text{if } \varphi, \text{ then } \psi]\!]^c$ just in case you are sure of $[\![\psi]\!]$ conditional on $[\![\varphi]\!]$. This is a virtue, since the global version of The Qualitative Thesis is subject to triviality results.

Why care about The Qualitative Thesis? There are at least three reasons. The first is that, given plausible assumptions, The Local Thesis entails The Qualitative Thesis. Those assumptions are:

- $S^{c,w}([\![\varphi]\!]^c)$ just in case $P_{c,w}([\![\varphi]\!]^c) = 1$; and
- $S^{c,w}_{[\![\varphi]\!]^c}([\![\psi]\!]^c)$ just in case $P_{c,w}([\![\psi]\!]^c | [\![\varphi]\!]^c) = 1$.

With these assumptions,The Qualitative Thesis becomes equivalent to

> **The Probability 1 Thesis**. If $P_{c,w}([\![\varphi]\!]^c) > 0$, then $P_{c,w}([\![\text{if } \varphi, \text{ then } \psi]\!]^c) = 1$ if and only if $P_{c,w}([\![\psi]\!]^c | [\![\varphi]\!]^c) = 1$

And The Probability 1 Thesis is a special case of The Local Thesis.

The second reason to care about The Qualitative Thesis concerns the well-attested observation that the indicative often behaves like the material conditional. Take, for instance, the famous or-to-if inference. Consider, for example, (1) and (2).

(1) Matt is either in Los Angeles or London.

(2) So, if Matt is not in Los Angeles, he is in London.

It's natural to infer (2) from (1) and The Qualitative Thesis can explain why. It ensures that the inference from (1) to (2) is a *reasonable inference* in the sense outlined in Stalnaker (1975): roughly, if the speakers in a given context come to accept (1), but leave open that Matt is not in Los Angeles, then they will also come to accept (2).[9]

[9] Note that it doesn't follow from the Qualitative Thesis that *whenever* the speakers accept (1), they are in a position to infer (2). They might be sure of (1) without leaving open that Matt is in Los Angeles, and the Qualitative Thesis is silent about that case. But, as Stalnaker points out, it is felicitous to assert (1) only if the context leaves open that Matt is not in Los Angeles, and so whenever (1) is felicitously asserted, the posterior context will entail that Matt is in Los Angeles or London, but leave open that Matt is in Los Angeles. This means that The Qualitative Thesis predicts that the speakers can infer (2) from (1) whenever (1) can be successfully asserted.

The final reason to care about The Qualitative Thesis is that it is needed to distinguish indicative conditionals from subjunctives. Consider Ernest Adams' famous minimal pair:

(3) If Oswald hadn't shot Kennedy, someone else would have.

(4) If Oswald didn't shoot Kennedy, someone else did.

(3) is a questionable claim about an alternative course of history, whereas (4) is straightforwardly true. The difference seems to be that while (4) is about how the world must have been, given what we now know, if Oswald wasn't the shooter, (3) is about how the world would have been had history taken a different course. When evaluating (3), we consider worlds that are incompatible with our evidence—ones where nobody shoots Kennedy. When evaluating (4), we only consider worlds that are compatible with what our current information—that someone shot Kennedy. A rigorous formulation of this constraint on indicatives was first given by [Stalnaker, 1975a]. Very roughly, Stalnaker's *Indicative Constraint* says that if the antecedent of an indicative conditional is compatible with our information, then one must evaluate its consequent relative to an antecedent world that is compatible with our information. In the next section, we show that, assuming Rational Monotonicity, The Qualitative Thesis is valid if and only if the Indicative Constraint holds. So, if we can't explain the difference between subjunctives and indicatives without the Indicative Constraint, we can't do so without The Qualitative Thesis, either.[10]

4 The Qualitative Thesis in the Standard Framework

Here we present a standard formal framework for thinking about The Qualitative Thesis, one that gives attitude ascriptions a Hintikka semantics and the conditional a variably strict semantics. We prove three results. The first characterizes the Qualitative Thesis on the set of *minimal* frames. The second characterizes it on the set of frames that obey Rational Monotonicity—the set of *minimal monotonic* frames. Finally, we prove that the Qualitative Thesis puts a significant constraint on the logic of sureness, entailing a principle we call *No Opposite Materials*.

[10] We can now say why we consider only our more qualified version of The Qualitative Thesis and not the more general one of Harper and Gärdenfors. The reason is that the or-to-if inference has counterexamples when the antecedent of the conditional is not left open. If it is raining, it follows that it is raining or snowing. However, the following does not seem like a reasonable inference:

a. It is raining.
b. #So if it isn't raining, it's snowing.

We do not want to say that *whenever* the speakers accept $\ulcorner\varphi$ or $\psi\urcorner$, they accept $\ulcorner$if$\neg\varphi$, then $\psi\urcorner$; rather, as Stalnaker is careful to point out, we want a version that is restricted to instances where the antecedent of the conclusion is possible.

4.1 Formal Framework and Stating the Qualitative Thesis

We begin by constructing a propositional modal language that we can use to describe what a subject is sure of. The set of sentences of the language $\mathcal{L}$ is the set of sentences generated by the following grammar:

- $\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \phi > \psi \mid S\phi \mid S_\phi\psi$

The propositional connectives $\supset$, $\equiv$, and $\vee$ are defined as usual. We read $S\phi$ as *the subject is sure of* ϕ and $S_\phi\psi$ as *the subject is sure of* ψ *conditional on* ϕ.

A *variably strict frame* $\mathcal{F}$ is a tuple $\langle W, R, R_{|\cdot}, f\rangle$. W is a non-empty set of worlds. R is a binary relation on W representing doxastic accessibility: wRw' means that w' is compatible with what the subject is sure of in w.[11] $R_{|\cdot}$ is a function from subsets of W to accessibility relations. Where **A** is an arbitrary subset of W, $wR_{|\mathbf{A}}w'$ means that w' is compatible with what the subject is sure of *conditional on* ***A*** in w.[12] Finally, f is the selection function that we use to interpret the conditional operator.

We interpret the language with a model $\mathcal{M} = \langle \mathcal{F}, V\rangle$. $\mathcal{F}$ is a variably strict frame and V a function from propositional variables to sets of worlds. We define truth in a model for non-conditional sentences as follows:

- $[\![p]\!]^w = 1$ iff $w \in V(p)$
- $[\![\neg\phi]\!]^w = 1$ iff $[\![\phi]\!]^w = 0$
- $[\![\phi \wedge \psi]\!]^w = 1$ iff $[\![\phi]\!]^w = [\![\psi]\!]^w = 1$

Truth in a model for conditional sentences is defined as follows:

> **Standard Variably Strict Semantics**. $[\![\phi > \psi]\!]^w = 1$ iff $f(w, [\![\phi]\!]) \subseteq [\![\psi]\!]$

where $[\![\phi]\!] = \{w : [\![\phi]\!]^w = 1\}$. This clause says that $\varphi > \psi$ is true at a world w just in case all of the selected φ-worlds at w are ψ-worlds.

Truth for the sureness operator S is defined in terms of the accessibility relation R.

> **Standard Hintikka Semantics**. $[\![S\phi]\!]^w = 1$ iff $\forall w' \in R(w) : [\![\phi]\!]^{w'} = 1$

Truth for the conditional sureness operator S_φ is defined in terms of the conditional accessibility relation $R_{|[\![\varphi]\!]}$.

[11] We use the term *doxastic accessibility* to mean compatibility with what the subject *is sure of*, not what she believes.

[12] We use upper case bold Roman letters pick out subsets of W. Occasionally we will be sloppy and use the connectives to denote set-theoretic operations on sets of worlds.

- $[\![S_\phi\psi]\!]^w = 1$ iff $\forall w' \in R_{|[\![\phi]\!]}(w) : [\![\psi]\!]^{w'} = 1$

$R_{|[\![\varphi]\!]}$ is the set of worlds consistent with what the subject is sure of conditional on φ.

We begin by defining the class of *minimal* variably strict frames. In a minimal frame, the selection function f obeys these constraints.

Success. $f(w, \mathbf{A}) \subseteq \mathbf{A}$

Minimality. If $w \in \mathbf{A}$, then $w \in f(w, \mathbf{A})$

Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $f(w, \mathbf{A}) \neq \emptyset$

Success and Minimality are standard assumptions.[13] Success says that the selected **A**-worlds at w must be **A**-worlds; it's needed to validate $\phi > \phi$. Minimality says that if w is an **A**-world, then it must be among the selected **A**-worlds at w; it's needed to validate Modus Ponens. Non-Vacuity says that if there are accessible **A**-worlds at w, then the set of selected **A**-worlds at w isn't empty. It's needed to validate a form of Conditional Non-Contradiction, specifically:

Weak Conditional Non-Contradiction. $\neg S\neg\phi \supset \neg((\phi > \psi) \wedge (\phi > \neg\psi))$

Weak Conditional Non-Contradiction says that if φ is a live possibility, then $\varphi > \psi$ and $\varphi > \neg\psi$ are not consistent. This is a standard—and desirable—principle in conditional logic.[14] In general, there is something very wrong with asserting both $\varphi > \psi$ and $\varphi > \neg\psi$.

In a minimal variably strict frame, $R_{|\cdot}$ obeys these constraints:

S-Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $R_{|\mathbf{A}}(w) \neq \emptyset$.

S-Success. $R_{|\mathbf{A}}(w) \subseteq \mathbf{A}$

Inclusion. $R(w) \cap \mathbf{A} \subseteq R_{|\mathbf{A}}(w)$

All three are standard assumptions from the literature on belief revision.[15,16] S-Non-Vacuity says that if **A** is consistent with what the subject is sure of, then the set of worlds compatible with what she is sure of conditional on **A** is not empty. S-Success says that the worlds compatible with

[13] See, for example, [Stalnaker, 1968] and [Lewis, 1973].

[14] Why not a stronger version of Conditional Non-Contradiction that just says $\varphi > \psi$ and $\phi > \neg\psi$ are not consistent? This stronger principle is inconsistent with Logical Implication, which says that $\varphi > \psi$ is always true when φ entails ψ. Weak Conditional Non-Contradiction, by contrast, is consistent with Logical Implication. See [Stalnaker, 1968] and [Lewis, 1973] for theories that validate a version of Conditional Non-Contradiction that is at least as strong as Weak Conditional Non-Contradiction.

[15] For example, S-Non-Vacuity, S-Success, and Inclusion are all axioms of the system in [Alchourrón et al., 1985].

[16] Given S-Non-Vacuity, S-Success, and Inclusion we could replace $R_{|\cdot}$ in our frames with a function from worlds to plausibility orderings, $\leq$, and then define $R_{|\mathbf{A}}(w)$ as the minimal **A**-worlds in the ordering at w. See [Grove, 1988] and [Pacuit, 2013] for details.

what the subject is sure of conditional on **A** are **A**-worlds. Inclusion says that the unconditionally accessible **A**-worlds are accessible conditional on **A**.[17]

A *minimal* frame is any variably strict frame $\langle W, R, R_{|\cdot}, f\rangle$ such that f obeys Success, Minimality, and Non-Vacuity, and $R_{|\cdot}$ obeys S-Non-Vacuity, S-Success, and Inclusion. A *minimal monotonic* variably strict frame is a minimal variably strict frame that meets the condition:

Conditional Identity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $R_{|\mathbf{A}}(w) = R(w) \cap \mathbf{A}$

Conditional Identity says that if there are unconditionally accessible **A**-worlds, then the accessible worlds conditional on **A** are just the unconditionally accessible **A**-worlds. Given our other constraints on $R_{|\cdot}$, Conditional Identity characterizes Rational Monotonicity, repeated below.

Rational Monotonicity. $(\neg S\neg\varphi \wedge S\psi) \supset S_{\varphi}\psi$

Fact 1. If $\mathcal{F}$ is a minimal variably strict frame, then Rational Monotonicity is valid on $\mathcal{F}$ iff $\mathcal{F}$ obeys Conditional Identity.

Rational Monotonicity is valid on the class of minimal monotonic frames. We provide the proof of Fact 1 and all subsequent facts in the appendices.

Now let's turn to formulating the Qualitative Thesis in this framework. Here's how we stated it in §3:

The Qualitative Thesis. For any world w and context c: If $\neg S^{c,w}([\![\neg\varphi]\!]^c)$, then: $S^{c,w}([\![$if φ, then $\psi]\!]^c)$ if and only if $S^{c,w}_{[\![\varphi]\!]^c}([\![\psi]\!]^c)$

The Qualitative Thesis says that if your total evidence is that of the speakers in c, then if you are not sure that $[\![\varphi]\!]$ is false, you are sure of the proposition expressed by ⌜if φ, then ψ⌝ in c just in case you are sure of $[\![\psi]\!]$ conditional on $[\![\varphi]\!]$.

As we're understanding things, a frame $\langle W, R, R_{|\cdot}, f\rangle$ represents an arbitrary context. The unconditional and conditional accessibility relations are those of the speakers in that context, and f is the selection function supplied by that context. This means that we can state The Qualitative Thesis as an object language principle, rather than stating it meta-linguistically as we did in §3.

Non-Monotonic QT. $\neg S\neg\phi \supset (S(\phi > \psi) \equiv S_{\phi}\psi)$

Given how we're thinking about frames, we are forced to interpret Non-Monotonic QT *locally*—specifically, as saying that if the speaker of a given context c leaves open $[\![\varphi]\!]$, then she is sure of

[17] Inclusion is trivially met when **A** is inconsistent with what the agent is sure of. Its purpose is to ensure that the set of accessible worlds conditional on **A** does not contain *more* information than would result from simply intersecting the unconditionally accessible worlds with **A**.

the proposition expressed by $\phi > \psi$ relative to the information in *her* context just in case she is sure of $[\![\psi]\!]$ conditional on $[\![\varphi]\!]$.

As we mentioned earlier, if we assume Rational Monotonicity, Non-Monotonic QT becomes equivalent to the following, perhaps more familiar, thesis.[18]

Material QT. $\neg S \neg \phi \supset (S(\phi > \psi) \equiv S(\phi \supset \psi))$

4.2 Characterizing Non-Monotonic QT and Material QT

As we mentioned before, we will first temporarily suspend the assumption of Rational Monotonicity to give a more general characterisation of The Qualitative Thesis in the standard framework. Then we will characterize it assuming Rational Monotonicity.

Consider the following constraint.

Union Constraint. If $R(w) \cap \mathbf{A} \neq \emptyset$, then: $\bigcup_{w' \in R(w)} f(w', \mathbf{A}) = R_{|\mathbf{A}}(w)$

The Union Constraint says that when we form the union of the selected **A**-worlds at each w' consistent with everything the subject is sure of at a world w, the resulting set is identical to the set of worlds consistent with what the subject is sure of conditional on **A** at w. The Union Constraint characterizes Non-Monotonic QT on the set of minimal variably strict frames:[19]

Fact 4. Non-Monotonic is valid on a minimal variably strict frame $\mathcal{F}$ if and only if $\mathcal{F}$ obeys the Union Constraint.

Going forward, we will neglect Non-Monotonic QT in favor of Material QT. We nonetheless characterize Non-Monotonic QT here for two reasons. Firstly, the result is novel and of independent interest. Secondly, Fact 4, and the epistemological consequences of it discussed in Appendix A.1, give the reader the resources to reconstruct our arguments in later sections without Rational Monotonicity. Note that since Material QT is the thesis we will be working with going forward, we will often refer to Material QT simply as QT.

Since we are assuming Rational Monotonicity, we want to characterize Material QT on the class of minimal *monotonic* frames—the minimal frames that obey Conditional Identity. Consider Stalnaker's *Indicative Constraint*:

Indicative Constraint. If $R(w) \cap \mathbf{A} \neq \emptyset$, then if $w' \in R(w)$, then $f(w', \mathbf{A}) \subseteq R(w)$.[20]

[18] See Facts 2 and 3 in the Appendix for the proof.

[19] The numbering of the facts here corresponds to that in the appendices.

[20] Versions of the Indicative Constraint are defended by [von Fintel, 1998], [Bacon, 2015], [Khoo, 2019], [Mandelkern and Khoo, ming] and [Mandelkern, 2019b].

The Indicative Constraint says that if **A** is compatible with what the speaker is sure of in a world w, then for any world w' that is compatible with what the speaker is sure of in w, the selected **A**-worlds at w' are a subset of the worlds compatible with what the subject is sure of at w.

In A.1, we prove that the Indicative Constraint characterizes Material QT on the set of minimal monotonic frames:

> **Fact 7**. Material QT is valid on a minimal monotonic variably strict frame $\mathcal{F}$ just in case $\mathcal{F}$ meets the Indicative Constraint.

Fact 7 is the characterization result we rely on from §5 onwards.

The proof of Fact 7 builds naturally on Fact 4. First we show that, on the set of minimal monotonic frames, the Union Constraint holds if and only if the Indicative Constraint does. Since we already know that Non-Monotonic QT and Material QT are equivalent on minimal monotonic frames, the chain of equivalences yields Fact 7.

4.3 No Opposite Materials

Finally, we turn to the epistemological consequences of The Qualitative Thesis. Consider the following property on frames:

> **No Opposite Materials**. For any two worlds w_1, w_2, if there's some w_3 such that $w_1 R w_3$ and $w_2 R w_3$, then, for any $\mathbf{A} \subseteq W$: if $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$, then there's no $\mathbf{C} \subseteq W$ such that $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$.

No Opposite Materials says that for certain pairs of worlds, and certain propositions **A**, you can't be sure of a material conditional $\mathbf{A} \supset \mathbf{C}$ at the first world and sure of the 'opposite' material conditional, $\mathbf{A} \supset \neg\mathbf{C}$, at the second. Which pairs of worlds? Any two worlds that see a world in common. And for which propositions? Any proposition that is consistent with what you're sure of at all three worlds.

In A.1 we prove Fact 10.

> **Fact 10**. A minimal monotonic variably strict frame $\mathcal{F}$ validates Material QT only if No Opposite Materials holds on $\mathcal{F}$.

In the next section we build on [Holguín, 2019] and show that No Opposite Materials is inconsistent with a plausible *margin for error* requirement on rational sureness.[21] Fact 10 tells us that Material QT entails No Opposite Materials. It follows that Material QT is itself inconsistent with the margin for error requirement.

[21] [Holguín, 2019] draws a very different moral from his argument, concluding that if you accept the margin for error principle you should reject The Qualitative Thesis. We think these can be reconciled.

5 No Opposite Materials and Margin for Error Principles

To illustrate the margin for error requirement, we begin with a case from Timothy Williamson.[22] Mr. Magoo is staring out the window at a tree some distance off, wondering how tall it is. Assuming his only sources of information are reflection and present perception of the tree, what should he believe? That depends on how tall the tree actually is. If the tree is 100 inches tall, Mr. Magoo's visual information rules out possibilities in which the tree is 200 inches tall, or so we can imagine. So it would be reasonable for Magoo to be sure that the tree is not 200 inches tall. On the other hand, Magoo's visual information does not rule out possibilities in which the tree is 101 inches tall; his eyesight is simply nowhere near that good. It would not be reasonable for Magoo to be sure that the tree is not 101 inches tall.

There's a general principle underlying these observations. Mr. Magoo's beliefs about the height of the tree are rational only if they leave a *margin for error*.[23] If the tree is n inches tall, a belief that the tree is not $n + 1$ inches tall does not leave a sufficiently wide margin for error; that belief is false in nearby worlds where the tree is slightly taller. On the other hand, a belief that the tree is not $n + 100$ inches tall does leave a sufficiently wide margin for error; that is true in nearby worlds where the tree is a bit taller.[24]

To state the margin for error requirement, we introduce a *margin for error frame* $\langle W, R\rangle$. W is a set of worlds representing possible tree heights. Where i is the height in inches of the tree in w, $W = \{w_i : i \in \mathbb{R} \text{ and } i > 0\}$. R is a binary doxastic accessibility relation on W: $w_i R w_j$ means that, in a world where the tree is i inches tall, it is compatible with everything Magoo is rationally sure of that the tree is j inches tall. R is defined as follows, relative to an arbitrarily chosen positive constant h.

Magoo's Margin. $w_i R w_j$ if and only if $|j - i| < h$.

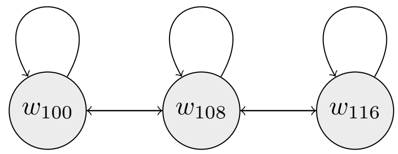
h is Magoo's margin for error; h is positive, for otherwise his discrimination would be perfect.

No Opposite Materials fails on every margin for error frame. To see this, suppose that $h = 10$, and consider three worlds in W: w_{100}, w_{108}, and w_{116}. Here is a diagram depicting Mr. Magoo's beliefs in these three worlds.

[22] See [Williamson, 2000].

[23] We will often use the term 'belief' because neither 'surety' nor 'sureness' sounds quite right (and 'surenessess' is even worse). But when we say 'Magoo's belief' we should be understood as talking about the state of being sure; and when we talk about 'Magoo's belief set' we should be understood as talking about the set of worlds compatible with what Magoo is sure of.

[24] Williamson introduces the margin for error principle as a requirement on knowledge, but as [Hawthorne and Magidor, 2009], [Hawthorne and Magidor, 2010] suggest, the principle is equally plausible for other attitudes. Hawthorne and Magidor focus on Stalnaker's attitude of *presupposition*, but similar considerations apply to rational sureness.

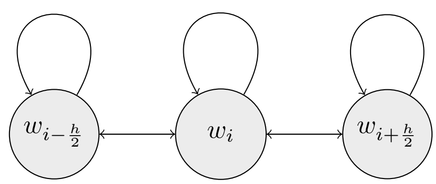


Mr. Magoo's belief worlds at w_{116} overlap with his belief worlds at w_{100}: w_{108} is consistent with what he is sure of in w_{116} and consistent with what he is sure of in w_{100}. Moreover, it's consistent with what Magoo is sure of at each world that the tree is either 100 inches tall or 116 inches tall. This means that the antecedent of No Opposite Materials is satisfied. The right and left worlds see a world in common, w_{108}. And the proposition that the tree is either 100 inches tall or 116 inches tall is consistent with what Magoo is sure of at all three worlds. But the consequent of No Opposite Materials is not satisfied. Since Magoo's margin for error is 10, w_{100} does not see w_{116} and w_{116} does not see w_{100}. As a result, Mr. Magoo is sure of 'opposite' material conditionals at w_{100} and w_{116}. At w_{100}, Mr. Magoo is sure that (5) is true; at w_{116}, Mr. Magoo is sure that (6) is true:

(5) $(116 \vee 100) \supset 100$

(6) $(116 \vee 100) \supset 116$

This shows that No Opposite Materials fails on every margin for error frame when $h = 10$. But the choice of 10 inches for h was arbitrary. It is not hard to see that No Opposite Materials will fail on every margin for error frame, regardless of the value of h.[25] Any such frame will contain, for some positive real number i, three worlds: w_i, $w_{i+\frac{h}{2}}$, and $w_{i-\frac{h}{2}}$.



[25] Williamson (2014) introduces more complex frames to model the margin for error requirement. These models treat worlds as ordered pairs $\langle j, k\rangle$, where j is the real height of the tree and k is the apparent height of the tree. He defines R as follows: $\langle j, k\rangle R\langle j', k'\rangle$ just in case (1) $k = k'$ and (2) $|j' - k| \leq |j - k| + h$. These frames validate No Opposite Materials. However, we think that this is merely an artifact of Williamson's simplifying assumption that appearances are luminous, which proponents of margin for requirements should ultimately reject: (1) says that one world sees another only if the apparent height of the tree is the same in the two worlds. If we replace (1) with a constraint (1$'$), which says that $\langle j, k\rangle R\langle j', k'\rangle$ only if $|k' - k| \leq c$, for some positive constant c, No Opposite Materials will no longer be valid. To see this, suppose $c = 6$ and $h = 10$. Then $\langle 108, 108\rangle$ and $\langle 114, 114\rangle$ see each other, and $\langle 108, 108\rangle$ and $\langle 102, 102\rangle$ see each other, but $\langle 102, 102\rangle$ does not see $\langle 114, 114\rangle$ and $\langle 114, 114\rangle$ does not see $\langle 102, 102\rangle$. In this model, the antecedent of No Opposite Materials is satisfied: $\langle 102, 102\rangle$ and $\langle 114, 114\rangle$ see a world in common, namely $\langle 108, 108\rangle$. And the proposition that the tree is either 102 inches tall or 114 inches tall is consistent with what Magoo is sure of at all three worlds. But the consequent of No Opposite Materials is not satisfied. At $\langle 102, 102\rangle$, Magoo is sure of the material conditional $(102 \vee 114) \supset 102$; at $\langle 114, 114\rangle$, Magoo is sure of the 'opposite' material conditional $(102 \vee 114) \supset 114$. Thanks to Simon Goldstein and Bernhard Salow for discussion.

The right and left worlds see a world in common, w_i. The proposition that the tree is either $i + \frac{h}{2}$ inches tall or $i - \frac{h}{2}$ inches tall is consistent with what Magoo is sure of at each world. So the antecedent of No Opposite Materials is satisfied. But the consequent is not. $w_{i-\frac{h}{2}}$ does not see $w_{i+\frac{h}{2}}$ and $w_{i+\frac{h}{2}}$ does not see $w_{i-\frac{h}{2}}$. This means that Mr. Magoo is sure of 'opposite' material conditionals at $w_{i-\frac{h}{2}}$ and $w_{i+\frac{h}{2}}$. At $w_{i-\frac{h}{2}}$, Magoo is sure of (7) and at $w_{i+\frac{h}{2}}$ he is sure of (8):

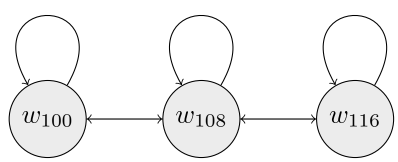
(7) $\quad (i + \frac{h}{2}) \vee (i - \frac{h}{2}) \supset (i - \frac{h}{2})$

(8) $\quad (i + \frac{h}{2}) \vee (i - \frac{h}{2}) \supset (i + \frac{h}{2})$

6 Enriching the Framework

In §4 we showed that The Qualitative Thesis entails No Opposite Materials in the standard variably strict framework. In §5 we showed that if we accept a margin-for-error requirement on rational sureness, we must reject No Opposite Materials. Putting these two things together, we conclude that if we accept the margin for error principle, we must reject The Qualitative Thesis.

We want to take a moment to explain the tension between No Opposite Materials and The Qualitative Thesis in a less formal, and hopefully more intuitive, way. Recall our three-world partial model of Mr. Magoo's sureness state. For ease of reference, we call this model *Williamson's Tree*.



Consider the proposition that the tree is either 100 inches tall or 116 inches tall ($100 \vee 116$). At w_{100}, Magoo is sure that (5) is true, and at w_{116} Magoo is sure that (6) is true.

(5) $\quad (100 \vee 116) \supset 100$

(6) $\quad (100 \vee 116) \supset 116$

Suppose The Qualitative Thesis holds at both w_{100} and w_{116}. Then Magoo is sure of the indicative conditionals (9) and (10) at w_{100} and w_{116}, respectively.

(9) $\quad (100 \vee 116) \ > \ 100$

(10) $\quad (100 \vee 116) \ > \ 116$

Remember that w_{108} is consistent with what Magoo is sure of at w_{100} and it is consistent with what he is sure of at w_{116}. So, (9) and (10) are both true at w_{108}. This is where we run into trouble. If (9) is true at w_{108}, the the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{100}\}$. If (10) it true at w_{108}, the selected $(100 \vee 116)$-worlds at w_{108} must be a subset of $\{w_{116}\}$.

But the selection function cannot meet both of these demands. The set of selected $(100 \vee 116)$-worlds at w_{108} can be a subset of $\{w_{100}\}$ and $\{w_{116}\}$ only if there are no selected $(100 \vee 116)$-worlds at w_{108}. But that would violate Non-Vacuity, which says that if an antecedent φ is doxastically possible at w, then the set of selected φ-worlds at w is not empty. We know that $(100 \vee 116)$ is consistent with what Magoo is sure of w_{108}: w_{108} sees w_{100} and w_{116}. So, by Non-Vacuity, the set of selected $(100 \vee 116)$-worlds at w_{108} is not empty.

In models that violate No Opposite Materials, The Qualitative Thesis places inconsistent demands on the selection function. Putting the problem this way suggests a solution. Instead of just one selection function, which we use to evaluate an indicative relative to just any belief state, we have multiple selection functions, indexed to different belief states. This will allow us to validate The Qualitative Thesis in models like Williamson's Tree. Instead of placing incompatible demands on one selection function, we place different demands on different selection functions. The selection function indexed to Magoo's belief state at w_{100} will satisfy a version of the Indicative Constraint stated in terms of Magoo's belief worlds at w_{100}. The selection function indexed to what Magoo is sure of at w_{116} will satisfy a version of the Indicative Constraint stated in terms of Magoo's belief worlds at w_{116}.

In the next section, we develop this proposal using the notion of a *local context* developed by Philippe Schlenker (though the central idea goes back to the pioneering works of [Stalnaker, 1975b], [Karttunen, 1974], and [Heim, 1992].)

7 Local Contexts and Shifty Selection Functions

We say that embedded conditionals are evaluated relative to their local contexts. When a conditional occurs under an attitude verb, the conditional is evaluated relative to the local context introduced by the attitude verb. We validate The Qualitative Thesis using a version of the Indicative Constraint. But importantly, our account is not subject to the problem of conflicting demands. That is because the selection function used to interpret the conditional is indexed to the conditional's local context. When the local context changes, the selection function does, too. In the rest of this section, we develop our theory. In §7.1, we say more about what local contexts are, describing how they have been used in theories of presupposition and epistemic modality.

In §7.2, we present our account: the *bounded, shifty* theory. In §7.3, we show how the theory validates The Qualitative Thesis while escaping the problem of conflicting demands.

7.1 What are Local Contexts?

Here's a standard idea. The interpretation of a sentence at a certain point in a conversation depends on the common commitments of the speakers at that point in the conversation. Starting in the early 1970s, theorists noticed that a sentence's *local* informational environment can also influence its interpretation. Specifically, how we interpret an expression in a sentence is partly determined by the information contained in the rest of the sentence.[26] The phenomenon of *presupposition projection* provides an illustration of this. Much contemporary research starts from the idea that a presupposition must be satisfied in the context in which it is uttered. But this won't do if by 'context' we mean the *global context*—the context of the conversation—modeled by a set of worlds representing the common commitments of the speakers. For consider (11):

(11) If Suzie used to smoke, she stopped smoking.

The consequent of (11) presupposes that Suzie used to smoke. But (11) can be felicitously uttered even when the speakers don't know that she used to smoke. This shows that the presupposition of the consequent of (11) need not be satisfied by the global context representing the common commitments of the speakers; instead, it only has to be satisfied relative to a kind of *local context* that includes the information contained in the antecedent of the conditional, that Suzie used to smoke. Following [Schlenker, 2009] and [Schlenker, 2010], we understand the local context of a clause of a sentence to be the unit of information that is *already available* for the interpretation of that clause. We will not need to be more precise about what local contexts are. What matters for our purposes is that they are bodies of information that are derived in a systematic way from the global context together with the material in the rest of the sentence.

The notion of a local context has proven useful in a different corner of semantic research, the study of epistemic modality. [Yalcin, 2007] noted the infelicity, both unembedded and embedded, of *epistemic contradictions*, sentences like:

(12) # It's raining and it might not be raining.

Epistemic contradictions are invariably defective. A natural explanation is that the epistemic modal conjunct takes for granted the information in the other conjunct: (12) sounds bad because *it might not be raining* is evaluated relative to an information state that entails that it is rain-

[26] See [Stalnaker, 1975b], [Karttunen, 1974], and [Heim, 1992].

ing. [Mandelkern, 2019a] uses the machinery of local contexts to refine this idea. On Mandelkern's *bounded theory* of epistemic modality, the domain of quantification for epistemic modals is limited by their local contexts. More precisely, an epistemic modal claim must be interpreted relative to a modal base that takes any world w to a subset of the modal's local context. This constraint, which Mandelkern calls the *Locality Constraint*, accounts for the infelicity of epistemic contradictions, embedded and unembedded.

Mandelkern's Locality Constraint bears a close similarity to the Indicative Constraint. The difference is that the Indicative Constraint is stated in terms of the *global context*—the set of worlds compatible with what the speakers believe—whereas the Locality Constraint is stated in terms of local contexts. In the next section, we propose to modify the Indicative Constraint so that it bears an even closer similarity to Mandelkern's Locality Constraint.[27]

7.2 The Localized Indicative Contraint and Shifty Selection Functions

We will assume a variably strict theory of the indicative conditional. Where κ is the conditional's local context, here's our semantic entry.

> **Bounded, Shifty Variably Strict Semantics**. $[\![\text{if } \varphi, \text{ then } \psi]\!]^{\kappa,w} = 1$ if and only if: $\forall w' \in f_\kappa(w, [\![\phi]\!]^\kappa) : [\![\psi]\!]^{\kappa,w'} = 1$

The Bounded, Shifty Variably Strict Semantics is similar to the Standard Variably Strict Semantics. The difference is that there is a new parameter—a local context parameter—and the selection function is indexed to that parameter. Since selection functions are indexed to local contexts, we can impose constraints on selection functions that make reference to local contexts. We propose to replace Stalnaker's Indicative Constraint with the following *Localized Indicative Constraint*:

> **Localized Indicative Constraint**. If $\mathbf{A} \cap \kappa \neq \emptyset$, then $\forall w' \in \kappa : f_\kappa(w', \mathbf{A}) \subseteq \kappa$

The Localized Indicative Constraint tells us that the selected antecedent worlds relative to a world w in the local context for the conditional must be a subset of the local context (so long as the antecedent is compatible with the local context).

With this new parameter, we restate the remaining constraints on the selection function.

> **Success.** $f_\kappa(w, \mathbf{A}) \subseteq \mathbf{A}$

> **Minimality**. If $w \in \mathbf{A}$, then $w \in f_\kappa(w, \mathbf{A})$.

[27] [Mandelkern, 2019b] has independently developed a version of the Localized Indicative Constraint. There are important differences between our constraint and Mandelkern's, however. The main difference is that Mandelkern's constraint is stated as a definedness condition on the interpretation of the conditional, whereas our constraint is stated as a constraint on the selection function.

Non-Vacuity. If $\kappa \cap \mathbf{A} \neq \emptyset$ then $f_\kappa(w, \mathbf{A}) \neq \emptyset$.

Success says that the selected **A**-worlds are a subset of **A**. Minimality says that if w is an **A**-world, then w is one of the selected **A**-worlds at w. We assume Success and Minimality for the same reasons as the standard framework does. Non-Vacuity says that if there are some **A**-worlds in κ, then the set of selected **A**-worlds at w is not empty. This constraint guarantees a *local* version of Weak Conditional Non-Contradiction: whenever there are φ-worlds in κ, at most one of $\phi > \psi$ and $\phi > \neg\psi$ can be true at a point of evaluation $\langle \kappa, w \rangle$.

We said that selection functions are indexed to local contexts and obey the Localized Indicative Constraint. The reason this matters, of course, is that local contexts are *shiftable*. In particular, they can be shifted by attitude predicates, such as *believe*, *want*, and, our focus in this paper, *is sure that*. Following [Schlenker, 2009], we assume that the local context introduced by an attitude predicate like *is sure that* at a world w is the set of worlds compatible with what the subject is sure of in w. Where R is a doxastic accessibility relation representing what an arbitrary subject is sure of and $R(w)$ is the set of worlds compatible with what that subject is sure of in w:

Shifty Hintikka Semantics. $[\![S\varphi]\!]^{\kappa,w} = 1$ if and only if: $\forall w' \in R(w) : [\![\varphi]\!]^{R(w),w'}$

Shifty Hintikka Semantics treats 'is sure that' as a necessity operator, just as the standard Hintikka semantics does. But now we've added a new parameter, a local context parameter, to the index. Shifty Hintikka Semantics says that attitude operators shift this parameter to $R(w)$, the set of worlds compatible with what the subject is sure of in w. This means that when we evaluate an attitude ascription like $\ulcorner$Magoo is sure that if φ, then $\psi\urcorner$ at a world w, we evaluate the embedded conditional relative to Magoo's belief state *at w*.[28] As we show in the next section, this is exactly what we need to validate The Qualitative Thesis without falling prey to the problem of conflicting demands.

7.3 Bounded, Shifty Indicatives and The Qualitative Thesis

We leave the proof to Appendix B, but here's an informal explanation of why QT, repeated below, is valid on our account.

QT. $\neg S\neg\varphi \supset (S(\varphi \supset \psi) \equiv S(\varphi > \psi))$

It will be useful to divide the thesis into two theses and take them in turn.

Indicative-to-Material. $\neg S\neg\varphi \supset (S(\varphi > \psi) \supset S(\varphi \supset \psi))$

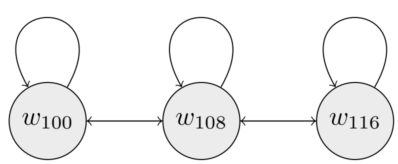
[28] One might notice a similarity between our view here and both information-sensitive and dynamic theories of attitudes and conditionals. See footnote 29 for further discussion.

Material-to-Indicative. $\neg S \neg\varphi \supset (S(\varphi \supset \psi) \supset S(\varphi > \psi))$

Begin with Indicative-to-Material. Suppose that, in an arbitrary world w, you are not sure of $\neg\varphi$ and you are sure of the indicative $\varphi > \psi$. Consider an arbitrary world w' that is compatible with what you are sure of in w. We know that $\varphi > \psi$ is true at w'. To show that $\varphi \supset \psi$ is true at w', suppose that φ is true at w'. Minimality tells us that if φ is true in w', then w' is among the selected φ-worlds at w'. Since $\varphi > \psi$ is true at w', it follows that ψ is true at w'. So, the material conditional $\varphi \supset \psi$ is true at w'. Since w' was chosen arbitrarily, we conclude that $\varphi \supset \psi$ is true at every world compatible with what you are sure of in w. You are sure of the material conditional $\varphi \supset \psi$ in w.

Turn to Material-to-Indicative. Suppose that, in an arbitrary world w, you are not sure of $\neg\varphi$ and you are not sure of the material conditional $\varphi \supset \psi$. Consider an arbitrary world w' that is compatible with what you are sure of in w. We want to show that $\varphi > \psi$ is true in w'. Since you are not sure that $\neg\varphi$ in w, the Localized Indicative Constraint tells us that selected all of the selected φ-worlds at w', *relative to your belief state in w*, are compatible with what you are sure of in w. Since you are sure of the material conditional $\varphi \supset \psi$ in w, all of these selected φ-worlds must be ψ-worlds. It follows that $\varphi > \psi$ is true at w' relative to your belief state in w. Since w' was chosen arbitrarily, we conclude that $\varphi > \psi$ is true, relative to your belief belief state in w, at every world compatible with what you are sure of in w. And that, according to **Shifty Hintikka Semantics**, is just what it takes for you to be sure of $\varphi > \psi$ in w.

That concludes our informal explanation of why The Qualitative Thesis is valid. The last thing to do is explain why we do not fall prey to the problem of conflicting demands in models where No Opposite Materials fails. So that we have everything in front of us, here is Williamson's Tree again.



Magoo is sure of the material conditional (5) in w_{100} and he is sure of the material conditional (6) in w_{116}.

(5) $(100 \vee 116) \supset 100$

(6) $(100 \vee 116) \supset 116$

In the standard variably strict framework, there is no way to guarantee that The Qualitative Thesis holds at both w_{100} and w_{116} without placing conflicting demands on the selection function at the overlap world w_{108}. To secure The Qualitative Thesis w_{100}, the selected (100 $\vee$ 116)-worlds at w_{108} must be a subset of $\{w_{100}\}$; otherwise $(100 \vee 116) > 100$ would be false at w_{108}, and so Magoo would not be sure of it at w_{100}. To secure The Qualitative Thesis at w_{116}, the selected (100 $\vee$ 116)-worlds at w_{108} must be a subset of $\{w_{116}\}$; otherwise $(100 \vee 116) > 116$ would be false at w_{108} Magoo would not be sure of it at w_{116}. The selection function cannot meet both of these demands on pain of violating Non-Vacuity.

In the bounded, shifty framework, by contrast, different belief states correspond to different selection functions. When we evaluate an indicative conditional relative to Magoo's belief state at w_{116}, we use one selection function; when we evaluate a conditional relative to his belief state at w_{100}, we use a different selection function. Consider (13) and (14):

(13) $[\![$Magoo is sure that: $100 \vee 116 > 100]\!]^{\kappa}$

(14) $[\![$Magoo is sure that: $100 \vee 116 > 116]\!]^{\kappa}$

Where R is an accessibility relation representing Magoo's beliefs, (13) is true at w_{100} just in case (15) is true at every world in $R(w_{100})$: w_{100} and w_{108}. (14) is true at w_{116} just in case (16) is true at every world in $R(w_{116})$: w_{108} and w_{116}.

(15) $[\![(100 \vee 116) > 100]\!]^{R(w_{100})}$

(16) $[\![(100 \vee 116) > 116]\!]^{R(w_{116})}$

But (15) and (16) do not place incompatible demands on the selection function at the overlap world w_{108}. (15) is true at w_{108} only if the selected $(100 \vee 116)$-world at w_{108}, *relative to Magoo's belief state at* w_{100}, is w_{100}, whereas (16) is true at w_{108} only if the selected $(100 \vee 116)$-world at w_{108}, *relative to Magoo's belief state at* w_{116}, is w_{116}. These are simply different demands on different selection functions, so there is no inconsistency.

8 Locality and Conditionals

We have argued for the following thesis about the interpretation of conditionals.

> **Conditional Locality**. When a conditional occurs under an attitude, the selection function used to evaluate the conditional is indexed to the local context introduced by the attitude and obeys the Localized Indicative Constraint.

We argued that we need Conditional Locality if we want to reconcile The Qualitative Thesis with the margin for error principle. Here we argue that Conditional Locality is independently motivated. The full range of data supports a thesis that is stronger than The Qualitative Thesis. We need Conditional Locality for this *Strong Qualitative Thesis*.[29]

8.1 Motivating The Strong Qualitative Thesis

When we constructed our formal framework in §4, we made a simplifying assumption, one that we continued to make throughout our discussion of the shifty framework. That assumption was that our formal language contains only one attitude operator, S. We also stipulated that S is coordinated with our conditional operator: S corresponds to the speaker of an arbitrary context, and $\varphi > \psi$ is interpreted relative to the information in that context. Of course, the stipulation that our formal language contains only one attitude operator is not a realistic one. What happens when we enrich the language with other attitude operators, corresponding to subjects whose information *differs* from that of the speaker of the context? We turn to this question now.

Where $\mathcal{A}$ is a finite set of names x, y, z, ... , we extend our modal propositional language $\mathcal{L}$ as follows:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \phi > \psi \mid S^x\phi \mid S^x_\phi\psi$$
$$x \in \mathcal{A}$$

Remember that we wanted our statement of QT to be a *local* thesis, saying that if the speaker of a given context leaves open $[\![\varphi]\!]$, then she is of the material conditional $[\![\varphi \supset \psi]\!]$ just in case she is sure of the corresponding indicative conditional that would be expressed relative to the information in her context. To state this local thesis, we need to distinguish the attitude operator corresponding to the speaker of the context from our other attitude operators. To do this, we add to our modal propositional language $\mathcal{L}$ a sureness operator corresponding to the speaker of c:

[29] Our account is not the only account that endorses something like Conditional Locality. Dynamic accounts of attitudes and conditionals, such as [Heim, 1992], [Dekker, 1993] and [Gillies, 2009], and informational accounts, such as [Yalcin, 2007], endorse something very similar. For example, on the informational theory, attitudes shift an information state parameter relevant for an embedded strict conditional in just the way that our attitudes shift the local context.

How does our view compare to these accounts? Both accounts can validate Strong QT. But they differ over the validity of or-to-if, which we formulate with an epistemic modal $\Diamond$ as:

Or-to-if. $\Diamond\neg\phi, \phi \vee \psi \models \neg\phi > \psi$

Dynamic and informational accounts validate or-to-if because they use non-classical consequence relations.

Since we use a classical consequence relation, we do not validate or-to-if. (However, we can account for the reliability of or-to-if *reasoning* using Stalnaker's notion of reasonable inference.) We think it is worth showing how to validate Strong QT in a static and classical variably strict framework, but we do not intend to argue at length here for this view over dynamic and informational accounts. However, we are inclined to think our account is right to *in*validate or-to-if. As [Santorio, 2019] shows, or-to-if is not probabilistically valid: it is easy to create counterexamples by focusing on cases where $[\![\phi]\!]$ by itself accounts for most of the probability of $[\![\phi \vee \psi]\!]$. If one is inclined to think, as we do, that valid inferences should preserve probability, then this feature of our account favors it over dynamic and informational theories. Thanks to Simon Goldstein for discussion.

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \phi > \psi \mid S^x\phi \mid S^x_\phi\psi \mid S^c\varphi \mid S^c_\phi\psi$$
$$x \in \mathcal{A}$$

Now we are in a position to restate QT in our extended modal propositional language.

QT. $\neg S^c\neg\phi \supset (S^c(\phi \supset \psi) \equiv S^c(\phi > \psi))$

The trouble is that QT doesn't give us everything we want. To see why, suppose I'm wondering where Matt is, and you have reason to believe that Alice knows. So you check with Alice and report back. Alice is sure that Matt is either in Los Angeles or London, but she's not sure which. So (17) is true in my context.

(17) Alice is sure that Matt is either in Los Angeles or London, but she's not sure which.

From (17), I infer (18).

(18) So, Alice is sure that if Matt is not in Los Angeles, he's in London.

The inference is reasonable, nay obligatory: if I accept (17), I must also accept (18). Observe that (19) seems to attribute to Alice an incoherent state of mind:

(19) # Alice is sure that Matt is either in Los Angeles or London, but she's not sure that he's in London if he's not in Los Angeles.

Just as you can't accept $\ulcorner\varphi$ or $\psi\urcorner$ while denying $\ulcorner$if $\neg\varphi$, then $\psi\urcorner$, you can't coherently describe *others* as accepting $\ulcorner\varphi$ or $\psi\urcorner$ while denying $\ulcorner$if $\neg\varphi$, then $\psi\urcorner$. In general, it seems, to say that someone is sure of the material conditional just is to say that she is sure of the corresponding indicative conditional.

But that is not what QT predicts. Suppose that I am the speaker in context c, the context relative to which we are interpreting our conditional operator $>$. Then QT says that if I am sure that Matt is either in Los Angeles or London, and I'm not sure that Matt is not in Los Angeles, then I am sure of the conditional expressed by *if Matt is not in Los Angeles, he's in London* relative to my information. But QT does not predict that (18) follows from (17) in my context. QT simply doesn't say anything at all about other subjects.

We want a version of the Qualitative Thesis that applies to *any* attitude operator in our language. Specifically:

Strong QT. $\neg S^x\neg\varphi \supset (S^x(\varphi \supset \psi) \equiv S^x(\varphi > \psi))$

Strong QT applies to all subjects, regardless of what context they're in, or what information they have. It predicts that if (17) is true in my context, then so is (18).

8.2 Strong QT without Conditional Locality

We've introduced Strong QT and argued that it is desirable. Here we argue that without Conditional Locality, Strong QT is untenable because it trivializes.

We'll use the standard variably strict framework from §4 as a representative example of a framework that rejects Conditional Locality. Let's first generalize the Standard Hintikka Semantics for our new language with multiple attitude operators, now also explicitly representing the role of context in our semantic evaluation function:

Standard Hintikka Semantics. $[\![S^s\phi]\!]^{c,w} = 1$ iff $\forall w' \in R_s(w) : [\![\phi]\!]^{c,w'} = 1$

Observe that the Standard Hintikka Semantics can be rewritten as follows.

(20) $[\![S^s\phi]\!]^{c,w} = 1$ iff $R_s(w) \subseteq [\![\phi]\!]^c$

Assume that an agent assigns probability 1 to a proposition **A** in a world w just in case the set of worlds compatible with what she is sure of in w is a subset of **A**. Where R_s is s's doxastic accessibility relation, we have:

(21) $P_{s,w}(\mathbf{A}) = 1$ iff $R_s(w) \subseteq \mathbf{A}$

(20) and (21) together entail the following:

Non-shifty Link. $[\![S^s\phi]\!]^{c,w} = 1$ iff $P_{s,w}([\![\phi]\!]^c) = 1$

The Non-Shifty Link and Strong QT together yield the Global Probability 1 Thesis:

The Global Probability 1 Thesis. For any subject s, context c, and world w: if $P_{s,w}([\![\phi]\!]^c) > 0$, then $P_{s,w}([\![\psi]\!]^c | [\![\phi]\!]^c) = 1$ iff $P_{s,w}([\![\phi > \psi]\!]^c) = 1$

Assuming Weak Conditional Non-Contradiction, The Global Probability 1 Thesis entails:

Global Preservation. For any subject s, context c, and world w, if $P_{s,w}([\![\varphi]\!]^c) > 0$ and $P_{s,w}([\![\psi]\!]^c) = 0$, then $P_{s,w}([\![\varphi > \psi]\!]^c = 0$.[30]

[30]Here is the proof that The Global Probability 1 Thesis entails Global Preservation.

1. $P_{s,w}([\![\varphi]\!]^c) > 0$ (assumption)
2. $P_{s,w}([\![\psi]\!]^c) = 0$ (assumption)
3. $P_{s,w}([\![\neg\psi]\!]^c) = 1$ (2, probability axioms)

But Global Preservation trivializes.

To see why, consider two subjects, s and s'. Suppose that the probability function of s' in w is s's probability function in w conditionalized $[\![\neg\psi]\!]^c$. Formally:

(22) $\quad P_{s',w} = P_{s,w}(\cdot|[\![\neg\psi]\!]^c)$

We make two assumptions: that s assigns positive probability to $[\![\varphi]\!]^c$ conditional on $[\![\neg\psi]\!]^c$ and that s assigns positive probability to $[\![\varphi > \psi]\!]^c$. Formally:

(23) $\quad P_{s,w}([\![\varphi]\!]^c|[\![\neg\psi]\!]^c) > 0.$

(24) $\quad P_{s,w}([\![\varphi > \psi]\!]^c) > 0$

(22) and (23), together with the probability axioms, yield the following facts:

(25) $\quad P_{s',w}([\![\phi]\!]^c) = P_{s,w}([\![\phi]\!]^c|[\![\neg\psi]\!]^c) > 0$

(26) $\quad P_{s',w}([\![\psi]\!]^c) = P_{s,w}([\![\psi]\!]^c|[\![\neg\psi]\!]^c) = 0$

Assuming Global Preservation, (25) and (26) entail (27):

(27) $\quad P_{s',w}([\![\phi > \psi]\!]^c) = Pr_{s',w}([\![\psi]\!]^c|[\![\phi]\!]^c) = 0$

But now remember that the probability function of s' is the probability function that results from conditionalizing s's probability function on $[\![\neg\psi]\!]^c$. This means that (27) entails (28):

(28) $\quad P_{s,w}([\![\phi > \psi]\!]^c|[\![\neg\psi]\!]^c) = 0$

And finally (24), (28), and the probability axioms give us (29).

(29) $\quad P_{s,w}([\![\psi]\!]^c|[\![\phi > \psi]\!]^c) = 1$

We have derived the consequence that any subject who assigns positive probability to $[\![\varphi]\!]^c$ conditional on $[\![\neg\psi]\!]^c$ and positive probability to the conditional $[\![\varphi > \psi]\!]^c$ is certain of $[\![\psi]\!]^c$ conditional on $[\![\varphi > \psi]\!]^c$. This result is absurd. I am not sure whether it is raining out or not. And I'm not sure whether we will have a picnic if it is sunny out. But I'm not sure that we will have

4. $P_{s,w}([\![\neg\psi]\!]^c|[\![\varphi]\!]^c) = 1$ (Ratio Formula)
5. $P_{s,w}([\![\varphi > \neg\psi]\!]^c) = 1$ (1,4 Global Probability 1 Thesis)
6. $P_{s,w}([\![\neg(\varphi > \psi)]\!]^c) = 1$ (1, 5, Weak Conditional Non-Contradiction)
7. $P_{s,w}([\![\varphi > \psi]\!]^c = 0$. (6, probability axioms)

a picnic on the hypothesis that *if it's sunny out, we're having a picnic*; learning this conditional doesn't settle for me whether or not we are going to have a picnic.[31]

The Global Probability 1 Thesis must be rejected. But remember that it follows from just two assumptions: Strong QT and the Non-Shifty Link. Moreover, the Non-Shifty Link follows from the Standard Hintikka Semantics, together with our assumption that a subject assigns probability 1 to a proposition **A** in w just in case the set of worlds compatible with what she is sure of in w is a subset of **A**. So, if she accepts this assumption, the proponent of the standard variably strict framework must reject Strong QT.

8.3 Strong QT with Conditional Locality

We have argued that Strong QT is untenable without Conditional Locality. With Conditional Locality, on the other hand, Strong QT is tenable. In particular, it is valid and non-trivializing in our bounded, shifty framework.

To see why Strong QT is valid, it's helpful to contrast the Indicative Constraint and the Localized Indicative Constraints:

> **Indicative Constraint.** If $R(w) \cap \mathbf{A} \neq \emptyset$, then if $w' \in R(w)$, then $f(w', \mathbf{A}) \subseteq R(w)$.
>
> **Localized Indicative Constraint**. If $\mathbf{A} \cap \kappa \neq \emptyset$, then if $w' \in \kappa : f_\kappa(w', \mathbf{A}) \subseteq \kappa$

On the standard, non-shifty variably strict semantics, which does not accept Conditional Locality, there is just one selection function. The Indicative Constraint coordinates this selection function with a *specific* accessibility relation—the accessibility relation relative to which we interpret S_c, the attitude operator corresponding to the speaker of the context. The selection function remains coordinated with that accessibility relation even when the conditional is embedded under other attitude operators that are interpreted relative to different accessibility relations.

Suppose, for example, that I am the speaker in c. And suppose that Alice, whose information differs from mine, is sure of the material conditional $[\![\varphi \supset \psi]\!]^c$. Does it follow that Alice is sure of the indicative conditional $[\![\varphi > \psi]\!]^c$? It doesn't. Suppose there is some world w that is compatible with what I am sure of and with what Alice is sure of. By the Indicative Constraint, the selected φ-worlds at w are a subset of the worlds compatible with what *I* am sure of, not the set of worlds compatible with what *Alice* is sure of. If there are worlds compatible with what I'm sure of where $[\![\varphi]\!]^c$ is true but $[\![\psi]\!]^c$ is not, then these selected φ-worlds may not be ψ-worlds, and in that case, $[\![\varphi > \psi]\!]^c$ will be false at w. Since w is compatible with what Alice is sure of, it follows that Alice is not of the indicative conditional $[\![\varphi > \psi]\!]^c$.

[31] This is a simplified version of Lewis's original triviality result targeting Stalnaker's Thesis. Our presentation of this result follows the presentation in [Russell and Hawthorne, 2016].

The Localized Indicative Constraint works differently. It picks the selected worlds from *whatever the local context for the conditional is*. More precisely, if there are ϕ-worlds in the local context, then the selected ϕ-worlds must be in the local context. The Shifty Hintikka Semantics ensures that for *any* sureness operator S^x, the local context for a conditional embedded under that operator is the set of worlds compatible with what x is sure of in the world of evaluation. The interpretation of the conditional is coordinated with the subject of the attitude clause. The Shifty Hintikka Semantics and the Localized Indicative Constraint combine to guarantee the validity of Strong QT. The precise explanation proceeds in just the same way as the explanation of why QT is valid given in §7.3.

Finally, turn to triviality. Without Conditional Locality, we are forced to reject Strong QT or face trivialization. If we accept Conditional Locality, and thus reject the Standard Hintikka Semantics in favor of the Shifty Hintikka Semantics, we needn't make this choice. That's because the Shifty Hintikka Semantics does not entail the Non-Shifty Link; instead, it entails the following *shifty* link.

Shifty Link. $[\![S^s\phi]\!]^{c,\kappa,w} = 1$ iff $P_{s,w}([\![\phi]\!]^{c,R(w)}) = 1$

The Shifty Link says that the sentence ⌜s is sure of ϕ⌝ is true in a context c just in case s has probability 1 in the proposition expressed by ϕ *relative to the local context introduced by the attitude predicate*.

To see why the Shifty Hintikka Semantics gives us the Shifty Link, note that can rewrite the semantic entry as follows.

(30) $[\![S^s\varphi]\!]^{c,\kappa,w} = 1$ if and only if: $R_s(w) \subseteq [\![\varphi]\!]^{c,R(w)}$

We also assume that an agent assigns probability 1 to a proposition **A** in a world w just in case the set of worlds compatible with what she is sure of in w is a subset of **A**. We repeat this assumption below.

(21) $P_{s,w}(\mathbf{A}) = 1$ iff $R_s(w) \subseteq \mathbf{A}$

The Shifty Link follows from (30) and (21). The Non-Shifty Link, on the other hand, fails.[32]

[32]Here is a simple counterexample. Assume there are just three worlds: w_1, where it rains and there is a picnic; w_2, where it rains and there is no picnic; and w_3, where it does not rain. Suppose w_1 and w_3 are compatible with what Billy is sure of in w_3 and w_2 and w_3 are compatible with what Alice is sure of in w_3. Now take a context where Alice is the speaker; so the global context for the conditional:

(31) If Matt isn't in Los Angeles, he's in London.

is what *Alice* is sure of. In other words, $[\![$If Matt isn't in Los Angeles, he's in London$]\!]^c$ is the same proposition as $[\![$If

The reason we avoid triviality is that The Shifty Link does not entail the Global Probability 1 Thesis; instead, it entails the Local Probability 1 Thesis:

> **The Local Probability 1 Thesis.** For any subject s, context c and world w: if $P_{s,w}([\![\varphi]\!]^{c,R_s(w)}) > 0$, then $P_{s,w}([\![\psi]\!]^{c,R_s(w)}|[\![\varphi]\!]^{c,R_s(w)}) = 1$ iff $P_{s,w}([\![\varphi > \psi]\!]^{c,R_s(w)}) = 1$

The Local Probability 1 Thesis is weaker than the Global Probability 1 Thesis. It doesn't say that just *anyone* must assign probability 1 to $[\![\varphi > \psi]\!]^{c,R_s(w)}$ just in case they assign probability 1 to $[\![\psi]\!]^{c,R_s(w)}$ conditional on $[\![\varphi]\!]^{c,R_s(w)}$. It says that s must assign probability 1 to $[\![\varphi > \psi]\!]^{c,R_s(w)}$ just in case s assigns probability 1 to $[\![\psi]\!]^{c,R_s(w)}$ conditional on $[\![\varphi]\!]^{c,R_s(w)}$. The equation holds only when the evidence determining the probability function and the evidence determining the interpretation of the conditional are identical.

This localization blocks the triviality result from earlier. Assume again that $P_{s',w}$ is the probability function that results from conditionalizing $P_{s,w}$ on $[\![\neg\psi]\!]^c$ and that $[\![\varphi]\!]^c$ and $[\![\neg\psi]\!]^c$ are compatible relative to $P_{s,w}$; that is, that s assigns positive probability to $[\![\varphi]\!]^c$ conditional on $[\![\neg\psi]\!]^c$.[33] The Global Probability 1 Thesis entails Global Preservation assuming Weak Conditional Non-Contradiction. And Global Preservation allows us to conclude that s' has probability 0 in $[\![\phi > \psi]\!]^c$. Since $P_{s',w}$ is $P_{s,w}$ conditionalized on $[\![\neg\psi]\!]^c$, we were able to conclude that s assigns probability 0 to $[\![\phi > \psi]\!]^c$ conditional on $[\![\neg\psi]\!]^c$, which, in turn, meant that s was certain of $[\![\psi]\!]^c$ conditional on $[\![\varphi > \psi]\!]^c$. But the Local Probability 1 Thesis does not have this consequence. That's because it does not require s' to have probability 0 in $[\![\phi > \psi]\!]^c$ in virtue of having probability 0 in $[\![\psi]\!]^c$ conditional on $[\![\varphi]\!]^c$; it only requires s' to have probability 0 in her *own* conditional, $[\![\phi > \psi]\!]^{c,R_{s'}(w)}$, when she has probability 0 in $[\![\psi]\!]^{c,R_{s'}(w)}$ conditional on $[\![\varphi]\!]^{c,R_{s'}(w)}$. This means that s can have non-extreme credence in $[\![\psi]\!]^c$ conditional on $[\![\phi > \psi]\!]^c$.

Matt isn't in Los Angeles, he's in London$]\!]^{c,R_{Alice}(w_3)}$. Billy does *not* assign this conditional probability 1 at w_3, for it is false at w_3: this conditional is about what *Alice* is sure of; and at w_3 Alice leaves open a world where Matt is in Paris, not London. So we have

(32) $P_{Billy,w_3}([\![$If Matt isn't in Los Angeles, he's in London$]\!]^c) \neq 1$

But in all worlds compatible with what Billy is sure of and where Matt isn't in Los Angeles, he's in Paris; so the following is true in c:

(33) Billy is sure that if Matt isn't in Los Angeles, he's in Paris.

[33] Note that, in the shifty framework, $[\![\phi]\!]^c$ is $[\![\phi]\!]^{c,\kappa_c}$, where κ_c is the set of worlds compatible with what the speaker in c is sure of.

9 Conclusion

This paper was centered around a dilemma. On the one hand, we saw that The Qualitative Thesis is a plausible thesis about the semantics of the conditional: it falls out of non-trivialising versions of Stalnaker's Thesis; it explains why indicatives often behave like materials; and it plays a central role in distinguishing indicatives from subjunctive conditionals. On the other hand, in standard frameworks, the Qualitative Thesis brings along substantial, nay unwelcome, epistemological commitments: it is incompatible with the margin for error principle, a plausible principle about the nature of rational sureness. We offered the local, shifty framework as a way out of that dilemma. We assumed that the interpretation of a conditional is sensitive to its local context; and we assumed that attitude operators shift that local context. The resulting theory can validate the Qualitative Thesis, even assuming a margin for error on rational sureness. Moreover, we showed that the central commitment of the theory, Conditional Locality, looks to be essential to validating a stronger, but also desirable, version of the Qualitative Thesis, with or without margin for error principles.

A The Qualitative Thesis in Standard Frameworks

A.1 The Variably Strict Framework

Our language $\mathcal{L}$ is the smallest set of sentences generated by the following grammar:

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \phi > \psi \mid S\phi \mid S_\phi \psi$$

A *variably strict (VS) frame* $\mathcal{F}$ for $\mathcal{L}$ is a tuple $\langle W, R, R_{|\cdot}, f\rangle$. W is a non-empty set of worlds. R is a binary relation on W. $R_{|\cdot}$ is a function from $\mathcal{P}(W)$ to $\mathcal{P}(W \times W)$. f is a function from W and $\mathcal{P}(W)$ to $\mathcal{P}(W)$.

Say that a minimal VS frame is one that obeys these constraints:

Success. $f(w, \mathbf{A}) \subseteq \mathbf{A}$

Minimality. If $w \in \mathbf{A}$, then $w \in f(w, \mathbf{A})$.

Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $f(w, \mathbf{A}) \neq \emptyset$.

S-Non-Vacuity If $R(w) \cap \mathbf{A} \neq \emptyset$ then $R_{|\mathbf{A}}(w) \neq \emptyset$.

S-Success $R_{|\mathbf{A}}(w) \subseteq \mathbf{A}$

Inclusion $R(w) \cap \mathbf{A} \subseteq R_{|\mathbf{A}}(w)$

Say that a *monotonic* minimal VS frame is a minimal VS frame that obeys:

Conditional Identity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $R_{|\mathbf{A}}(w) = R(w) \cap \mathbf{A}$

We recursively define truth at point in W:

$[\![p]\!]^w = 1$ iff $w \in V(p)$

$[\![\neg\phi]\!]^w = 1$ iff $[\![\phi]\!]^w = 0$

$[\![\phi \wedge \psi]\!]^w = 1$ iff $[\![\phi]\!]^w = [\![\psi]\!]^w = 1$

$[\![\phi > \psi]\!]^w = 1$ iff $f(w, [\![\phi]\!]) \subseteq [\![\psi]\!]$

$[\![S\phi]\!]^w = 1$ iff $\forall w' \in R(w) : [\![\phi]\!]^{w'} = 1$

$[\![S_\phi \psi]\!]^w = 1$ iff $\forall w' \in R_{|[\![\phi]\!]}(w) : [\![\psi]\!]^{w'} = 1$

where $[\![\phi]\!] = \{w : [\![\phi]\!]^w = 1\}$.

Recall:

Rational Monotonicity. $(\neg S\neg\phi \wedge S\psi) \supset S_\phi\psi$

Fact 1. If $\mathcal{F}$ is a minimal VS frame, Rational Monotonicity is valid iff $\mathcal{F}$ obeys Conditional Identity.

Proof. $\Rightarrow$: Suppose $\mathcal{F}$ is a minimal VS frame that does not obey Conditional Identity. Then there is some $\mathbf{A}$ and w such that $R(w) \cap \mathbf{A} \neq \emptyset$ but $R_{|\mathbf{A}}(w) \neq R(w) \cap \mathbf{A}$. Since Inclusion holds on a minimal VS frame, it must be that there is some w' such that $w' \in R_{|\mathbf{A}}(w)$ but $w' \notin R(w)$. Let $V(p) = \mathbf{A}$ and $V(q) = R(w)$. Then $[\![(\neg S\neg p \wedge Sq) \supset S_p q]\!]^w = 0$.

$\Leftarrow$: Suppose $\mathcal{F}$ is a minimal VS frame and $(\neg S\neg\phi \wedge S\psi) \supset S_\phi\psi$ is not valid. Then $R(w) \cap [\![\phi]\!] \neq \emptyset$, $R(w) \subseteq [\![\psi]\!]$ but $R_{|\mathbf{A}}(w) \nsubseteq [\![\psi]\!]$. So $R_{|\mathbf{A}}(w) \neq R(w) \cap \mathbf{A}$ and Conditional Identity fails.

Fact 2. If $\mathcal{F}$ is a minimal monotonic VS frame, $\neg S\neg\phi \supset (S_\phi\psi \equiv S(\phi \supset \psi))$ is valid on $\mathcal{F}$.

Proof. Suppose the above schema were invalid on a minimal VS frame $\mathcal{F}$. Then one of two cases obtains: i) for some w: $[\![\neg S\neg\phi]\!]^w = [\![S_\phi\psi]\!]^w = 1$ and $[\![S(\phi \supset \psi)]\!]^w]\!] = 0$; or ii) for some w: $[\![\neg S\neg\phi]\!]^w = [\![S(\phi \supset \psi)]\!]^w = 1$ and $[\![S_\phi\psi]\!]^w = 0$. In case i), this means $R(w) \cap [\![\phi]\!] \neq \emptyset$, $R(w) \cap [\![\phi]\!] \nsubseteq [\![\psi]\!]$ and $R_{|[\![\phi]\!]}(w) \subseteq [\![\psi]\!]$. Thus Monotonicity fails and so $\mathcal{F}$ is not a monotonic minimal VS frame. Similarly, if case ii) obtains then $R(w) \cap [\![\phi]\!] \neq \emptyset$, $R(w) \cap [\![\phi]\!] \subseteq [\![\psi]\!]$ and $R_{[\![\phi]\!]}(w) \nsubseteq [\![\phi]\!]$, so again Monotonicity fails.

Recall from section 4 our object language versions of the The Qualitative Thesis:

Non-Monotonic QT $\neg S\neg\phi \supset (S(\phi > \psi) \equiv S_\phi\psi)$

QT $\neg S\neg\phi \supset (S(\phi > \psi) \equiv S(\phi \supset \psi))$

Fact 3. If $\mathcal{F}$ is a minimal monotonic frame, then Non-Monotonic QT is valid on $\mathcal{F}$ iff QT is valid on $\mathcal{F}$.

Proof. Immediate from Fact 2.

Now recall the following principle:

Union Constraint. If $R(w) \cap \mathbf{A} \neq \emptyset$, then: $\bigcup_{w' \in R(w)} f(w', \mathbf{A}) = R_{|\mathbf{A}}(w)$

Fact 4. Non-Monotonic QT is valid on a minimal VS frame $\mathcal{F}$ iff $\mathcal{F}$ obeys the Union Constraint.

Proof. We split the Qualitative Thesis into two principles:

Non-Monotonic QT$_\Leftarrow$ $\neg S\neg\phi \supset (S_\phi\psi \supset S(\phi > \psi))$

Non-Monotonic QT$_\Rightarrow$ $\neg S\neg\phi \supset (S(\phi > \psi) \supset S_\phi\psi)$

We first show that each of these principles is characterised by the following properties on frames:

Superset Constraint: If $R(w) \cap \mathbf{A} \neq \emptyset$, then: $\bigcup_{w' \in R(w)} : f(w', \mathbf{A}) \supseteq R_{|\mathbf{A}}(w)$

Subset Constraint: If $R(w) \cap \mathbf{A} \neq \emptyset$, then: $R_{|\mathbf{A}}(w) \supseteq \bigcup_{w' \in R(w)} f(w', \mathbf{A})$

Lemma 1. QT$_{\Leftarrow}$ is valid on a minimal VS frame $\mathcal{F}$ iff $\mathcal{F}$ meets the Subset Constraint.

Proof. $\Rightarrow$: Suppose a minimal VS $\mathcal{F}$ does not meet the Subset Constraint. Then, for some $\mathbf{A}$ and w, (i) $R(w) \cap \mathbf{A} \neq \emptyset$ and (ii) there's a $w' \in R(w)$ such that there's a $w'' \in f(w', \mathbf{A})$ that is not in $R_{|\mathbf{A}}(w)$. We can then build a model on $\mathcal{F}$ that invalidates Non-Monotonic QT$_{\Leftarrow}$. Let $V(p) = \mathbf{A}$ and $V(q) = \{w''\}$. $\neg S \neg p$ and $S_p \neg q$ are true at w; but $S(p > \neg q)$ is false at w.

$\Leftarrow$: Suppose Non-Monotonic QT$_{\Leftarrow}$ is not valid on a minimal VS $\mathcal{F}$. Then in some model on $\mathcal{F}$ there is some world w s.t. $[\![\neg S \neg \phi]\!]^w = [\![S_\phi \psi]\!]^w = 1$ and $[\![S(\phi > \psi)]\!]^w = 0$. So $R(w) \cap [\![\phi]\!] \neq \emptyset$, $R_{|[\![\phi]\!]}(w) \subseteq \psi$ and for some $w' \in R(w) : f(w', [\![\phi]\!]) \nsubseteq [\![\psi]\!]$. So for some $w' \in R(w)$: $f(w', [\![\phi]\!]) \nsubseteq R_{|[\![\phi]\!]}(w)$. So the Subset Constraint fails on $\mathcal{F}$.

Lemma 2. Non-Monotonic QT$_{\Rightarrow}$ is valid in a minimal VS frame $\mathcal{F}$ iff $\mathcal{F}$ meets the Superset Constraint.

Proof. $\Rightarrow$: Suppose a minimal VS $\mathcal{F}$ does not meet the Superset Constraint. Then there's a $\mathbf{A}$ and w such that $R(w) \cap \mathbf{A} \neq \emptyset$ and such that there's a $w' \in R_{|\mathbf{A}}(w) : w' \notin \bigcup_{w'' \in R(w)} f(w'', \mathbf{A})$. We can build a model on $\mathcal{F}$ where Non-Monotonic QT$_{\Rightarrow}$ fails. Let $V(p) = \mathbf{A}$ and $V(q) = \{w''\}$. $\neg S \neg p$ and $S(p > \neg q)$ are both true at w; but $S_p \neg q$ is false.

$\Leftarrow$: Suppose Non-Monotonic QT$_{\Rightarrow}$ is not valid on a minimal VS $\mathcal{F}$. Then on some model on $\mathcal{F}$ there is some world w such that $[\![\neg S \neg \phi]\!]^w = [\![S(\phi > \psi)]\!]^w = 1$ but $[\![S_\phi \psi]\!]^w = 0$. So, at w, $R(w) \cap [\![\phi]\!] \neq \emptyset$, for all $w' \in R(w) : f(w', [\![\phi]\!]) \subseteq [\![\psi]\!]$ but $R_{|[\![\phi]\!]}(w) \nsubseteq \psi$. So there is a $w' \in R_{|[\![\phi]\!]}(w)$ such that for all $w'' \in R(w) : w' \notin f(w'', [\![\phi]\!])$. So the Superset Constraint fails.

Fact 4 follows immediately from Lemmas 1 and 2.

Fact 5. If $\mathcal{F}$ is a minimal monotonic VS frame, then $\mathcal{F}$ obeys the Union Constraint if and only if $\mathcal{F}$ obeys the Indicative Constraint.

$\Rightarrow$: Suppose a minimal monotonic VS $\mathcal{F}$ does not obey the Indicative Constraint. Then for some world w and some proposition $\mathbf{A}$, $R(w) \cap \mathbf{A} \neq \emptyset$ and there's a world $w' \in f(w, \mathbf{A})$ such that $w' \notin R(w)$. If $w' \in f(w, \mathbf{A})$ and $w' \notin R(w)$, then $f(w, \mathbf{A}) \nsubseteq R(w)$. And if $f(w, \mathbf{A}) \nsubseteq R(w)$, then $f(w, \mathbf{A}) \nsubseteq R(w) \cap \mathbf{A}$—that is, $f(w, \mathbf{A}) \nsubseteq R_{|\mathbf{A}}(w)$. It follows that $\bigcup_{w' \in R(w)} f(w', \mathbf{A}) \nsubseteq R_{|\mathbf{A}}(w)$: Union Constraint is not valid in $\mathcal{F}$.

$\Leftarrow$: It suffices to show that the Indicative Constraint entails the Subset and Superset Constraints. The Subset Constraint follows from Conditional Identity and Minimality. Minimality tells us that for all $w' \in R(w) \cap \mathbf{A}$: $w' \in f(w', \mathbf{A})$. It follows that for all $w' \in R(w) \cap \mathbf{A} : w' \in \bigcup_{w'' \in R(w)} f(w'', \mathbf{A})$, which is just to say that $R(w) \cap \mathbf{A} \subseteq \bigcup_{w'' \in R(w)} f(w'', \mathbf{A})$. Conditional Identity tells us that $R_{|\mathbf{A}}(w) = R(w) \cap \mathbf{A}$. So, $R_{|\mathbf{A}}(w) \subseteq \bigcup_{w'' \in R(w)} f(w'', \mathbf{A})$.

The Superset Constraint follows from the Indicative Constraint, Conditional Identity, and Success. The Indicative Constraint says that for all $w' \in R(w)$, $f(w', \mathbf{A}) \subseteq R(w)$. Success tells us that $f(w', \mathbf{A}) \subseteq \mathbf{A}$. Putting these two together, we conclude that $f(w', \mathbf{A}) \subseteq R(w) \cap \mathbf{A}$, for all $w' \in R(w)$. Since $R_{|\mathbf{A}}(w) = R(w) \cap \mathbf{A}$ (by Conditional Identity), we know that $f(w', \mathbf{A}) \subseteq R_{|\mathbf{A}}(w)$, for all $w' \in R(w)$. It follows that $\bigcup_{w' \in R(w)} f(w', \mathbf{A}) \subseteq R_{|\mathbf{A}}(w)$.

Fact 6. Non-Monotonic QT is valid on a minimal monotonic VS frame $\mathcal{F}$ just in case $\mathcal{F}$ meets the Indicative Constraint.

Proof. Immediate from Facts 4 and 5.

Fact 7. QT is valid on a minimal monotonic VS frame $\mathcal{F}$ just in case $\mathcal{F}$ meets the Indicative Constraint.

Proof. Immediate from Facts 3 and 6.

Consider:

> **Only Consistent Learning**. For any two worlds w_1, w_2, if there's some w_3 such that $w_1 R w_3$ and $w_2 R w_3$, then, for any $\mathbf{A} \subseteq W$: if $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$, then $R_{|\mathbf{A}}(w_1) \cap R_{|\mathbf{A}}(w_2) \neq \emptyset$.

And recall:

> **No Opposite Materials**. For any two worlds w_1, w_2, if there's some w_3 such that $w_1 R w_3$ and $w_2 R w_3$, then, for any $\mathbf{A} \subseteq W$: if $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$, then there's no $\mathbf{C} \subseteq W$ such that $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$.

Fact 8. If $\mathcal{F}$ is a minimal monotonic VS frame, $\mathcal{F}$ obeys Only Consistent Learning iff $\mathcal{F}$ obeys No Opposite Materials.

Proof. Pick an arbitrary w_1, w_2, w_3 and $\mathbf{A}$ for which the antecedent of both Only Consistent Learning and No Opposite Materials holds. By Conditional Identity, $R_{|\mathbf{A}}(w_1) = R(w_1) \cap \mathbf{A}$ and $R_{|\mathbf{A}}(w_2) = R(w_2) \cap \mathbf{A}$. So $R_{|\mathbf{A}}(w_1) \cap R_{|\mathbf{A}}(w_2) \neq \emptyset$ iff $(R(w_1) \cap \mathbf{A}) \cap (R(w_2) \cap \mathbf{A}) \neq \emptyset$ iff there's no $\mathbf{C}$ such that $R(w_1) \subseteq \mathbf{A} \supset \mathbf{C}$ and $R(w_2) \subseteq \mathbf{A} \supset \neg\mathbf{C}$.

Fact 9. A minimal VS frame $\mathcal{F}$ validates Non-Monotonic QT only if Only Consistent Learning holds on $\mathcal{F}$.

Proof. Suppose that Only Consistent Learning fails on $\mathcal{F}$. Then there are w_1 w_2, w_3 and **A** such that (i) $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$ but (ii) $R_{|\mathbf{A}}(w_1) \cap R_{|\mathbf{A}}(w_2) = \emptyset$. Now, for contradiction, suppose that Non-Monotonic QT is valid on $\mathcal{F}$. Since that is so iff $\mathcal{F}$ obeys the Union Constraint, we have that $f(\mathbf{A}, w_3) \subseteq R_{|\mathbf{A}}(w_1)$ and $f(\mathbf{A}, w_3) \subseteq R_{|\mathbf{A}}(w_2)$. But by supposition $R_{|\mathbf{A}}(w_1) \cap R_{|\mathbf{A}}(w_2) = \emptyset$. So $f(\mathbf{A}, w_3) = \emptyset$. But, since $R(w_3) \cap \mathbf{A} \neq \emptyset$, this contradicts Non-Vacuity.

Fact 10. If $\mathcal{F}$ is a minimal monotonic VS frame, then QT is valid on $\mathcal{F}$ only if No Opposite Materials holds on $\mathcal{F}$.

Proof. Immediate from Facts 9 and 10.

A.2 The Qualitative Thesis in a Strict Framework

Our language $\mathcal{L}$ is as before. A *strict frame* $\mathcal{F}$ for $\mathcal{L}$ is a tuple $\langle W, R, R_{|\mathbf{A}}, h\rangle$. W is a non-empty set of worlds. R is a binary relation on W. $R_{|\mathbf{A}}$ is a function from $\mathcal{P}(W)$ to $\mathcal{P}(W \times W)$. h is a function from W to $\mathcal{P}(W)$.

A minimal strict frame obeys S-Success, S-Non Vacuity, Inclusion and the following constraints on h:

Strict Success. $w \in h(w)$

Strict Non-Vacuity. If $R(w) \cap \mathbf{A} \neq \emptyset$ then $h(w) \cap \mathbf{A} \neq \emptyset$.

A minimal monotonic strict frame is a minimal strict frame that obeys Conditional Identity.

Our truth-conditions for the conditional are:

$$[\![\phi > \psi]\!]^w = 1 \text{ iff } (h(w) \cap [\![\phi]\!]) \subseteq [\![\psi]\!]$$

All of our other clauses remain the same as before.

Consider:

Strict Union Constraint. If $R(w) \cap \mathbf{A} \neq \emptyset$, then: $\bigcup_{w' \in R(w)} (h(w') \cap \mathbf{A}) = R_{|\mathbf{A}}(w)$

Fact 11. Non-Monotonic QT is valid on a minimal strict frame $\mathcal{F}$ iff $\mathcal{F}$ meets the Strict Union Constraint.

Proof. Again we split Non-Monotonic QT into Non-Monotonic QT$_{\Leftarrow}$ and Non-Monotonic QT$_{\Rightarrow}$ and characterise them. Consider:

Strict Superset Constraint: If $R(w) \cap \mathbf{A} \neq \emptyset$, then: $\bigcup_{w' \in R(w)}(h(w') \cap \mathbf{A}) \supseteq R_{|\mathbf{A}}(w)$

Strict Subset Constraint: If $R(w) \cap \mathbf{A} \neq \emptyset$, then: $R_{|\mathbf{A}}(w) \supseteq \bigcup_{w' \in R(w)}(h(w') \cap \mathbf{A})$

Lemma 3. Non-Monotonic QT$_{\Leftarrow}$ is valid on a a minimal strict frame $\mathcal{F}$ iff $\mathcal{F}$ meets the Strict Subset Constraint.

Proof. $\Rightarrow$: Suppose a minimal strict frame $\mathcal{F}$ does not meet the Subset Constraint. Then, for some $\mathbf{A}$ and w, (i) $R(w) \cap \mathbf{A} \neq \emptyset$ and (ii) there's a $w' \in R(w)$ such that there's a $w'' \in h(w') \cap \mathbf{A}$ that is not in $R_{|\mathbf{A}}(w)$. We can then build a model on $\mathcal{F}$ that invalidates Non-Monotonic QT$_{\Leftarrow}$. Let $V(p) = \mathbf{A}$ and $V(q) = \{w''\}$. $\neg S \neg p$ and $S_p \neg q$ are true at w; but $S(p > \neg q)$ is false at w.

$\Leftarrow$: Suppose Non-Monotonic QT$_{\Leftarrow}$ is not valid on a minimal strict frame $\mathcal{F}$. Then in some model on $\mathcal{F}$ there is some world w s.t. $[\![\neg S \neg \phi]\!]^w = [\![S_\phi \psi]\!]^w = 1$ and $[\![S(\phi > \psi)]\!]^w = 0$. So $R(w) \cap [\![\phi]\!] \neq \emptyset$, $R_{|[\![\phi]\!]}(w) \subseteq \psi$ and for some $w' \in R(w) : (h(w') \cap [\![\phi]\!]) \nsubseteq [\![\psi]\!]$. So for some $w' \in R(w) : (h(w') \cap [\![\phi]\!]) \nsubseteq R_{|[\![\phi]\!]}(w)$. So the Subset Constraint fails on $\mathcal{F}$.

Lemma 4. Non-Monotonic QT$_{\Rightarrow}$ is valid in a minimal strict frame $\mathcal{F}$ iff $\mathcal{F}$ meets the Strict Superset Constraint.

Proof. $\Rightarrow$: Suppose a minimal strict frame $\mathcal{F}$ does not meet the Strict Superset Constraint. Then there's a $\mathbf{A}$ and w such that $R(w) \cap \mathbf{A} \neq \emptyset$ and such that there's a $w' \in R_{|\mathbf{A}}(w) : w' \notin \bigcup_{w'' \in R(w)}(h(w'') \cap \mathbf{A})$. We can build a model on $\mathcal{F}$ where Non-Monotonic QT$_{\Rightarrow}$ fails. Let $V(p) = \mathbf{A}$ and $V(q) = \{w''\}$. $\neg S \neg p$ and $S(p > \neg q)$ are both true at w; but $S_p \neg q$ is false.

$\Leftarrow$: Suppose Non-Monotonic QT$_{\Rightarrow}$ is not valid in a minimal strict frame $\mathcal{F}$. Then in some model on $\mathcal{F}$ there is some world w such that $[\![\neg S \neg \phi]\!]^w = [\![S(\phi > \psi)]\!]^w = 1$ but $[\![S_\phi \psi]\!]^w = 0$. So, at w, $R(w) \cap [\![\phi]\!] \neq \emptyset$, for all $w' \in R(w) : h(w') \cap [\![\phi]\!] \subseteq [\![\psi]\!]$ but $R_{|[\![\phi]\!]}(w) \nsubseteq \psi$. So there is a $w' \in R_{|[\![\phi]\!]}(w)$ such that for all $w'' \in R(w) : w' \notin h(w'') \cap [\![\phi]\!]$. So the Superset Constraint fails.

Fact 11 is immediate from Lemmas 3 and 4.

Now consider:

Strict Indicative Constraint. If $R(w) \cap \phi \neq \emptyset$ then for all $w' \in R(w) : (h(w') \cap \phi) \subseteq R(w)$

Fact 12. If $\mathcal{F}$ is a minimal monotonic strict frame, then $\mathcal{F}$ obeys the Strict Indicative Constraint iff it obeys the Strict Union Constraint.

Proof. $\Leftarrow$: If $\mathcal{F}$ is a minimal monotonic strict frame, then if $R(w) \cap \mathbf{A} \neq \emptyset$ then $R_{|\mathbf{A}}(w) = R(w) \cap \mathbf{A}$. So the Strict Union Constraint holds iff if $R(w) \cap \mathbf{A}$, then $\bigcup_{w' \in R(w)}(h(w') \cap \mathbf{A}) =$

$(R(w) \cap \mathbf{A})$. So if the Strict Union Constraint holds, then if $R(w) \cap \mathbf{A}$ then for all $w' \in R(w)$: $(h(w') \cap \mathbf{A}) \subseteq R(w)$.

$\Rightarrow$: It suffices to show that if the Strict Indicative Constraint holds on a minimal strict $\mathcal{F}$ then so do both the Strict Subset and Strict Superset Constraints. First take the Strict Superset Constraint. Suppose that the Strict Indicative Constraint holds and suppose $R(w) \cap \mathbf{A} \neq \emptyset$. By Strict Success, for every $w'' \in R(w) \cap \mathbf{A}$, $w'' \in h(w'')$. So then $\bigcup_{w' \in R(w)}(h(w') \cap \mathbf{A}) \supseteq R(w) \cap \mathbf{A}$. By Conditional Identity it follows that $\bigcup_{w' \in R(w)}(h(w') \cap \mathbf{A}) \supseteq R_A(w)$.

Now take the Strict Subset Constraint. Suppose $R(w) \cap \mathbf{A} \neq \emptyset$. By the Strict Indicative Constraint for all $w' \in R(w) : h(w) \cap \mathbf{A} \subseteq R(w)$. But then for all $w' \in R(w) : h(w) \cap \mathbf{A} \subseteq R(w) \cap \mathbf{A}$. Since Conditional Identity holds, for all $w' \in R(w) : h(w) \cap \mathbf{A} \subseteq R_{|\mathbf{A}}(w)$. So $R_{|\mathbf{A}}(w) \supseteq \bigcup_{w' \in R(w)}(h(w') \cap \mathbf{A})$.

Fact 13. A minimal strict frame $\mathcal{F}$ validates the Qualitative Thesis only if Only Consistent Learning holds on that frame.

Proof. Suppose that Only Consistent Learning fails on $\mathcal{F}$. Then there are w_1, w_2, w_3 and $\mathbf{A}$ such that (1) $R(w_1) \cap \mathbf{A} \neq \emptyset$, $R(w_2) \cap \mathbf{A} \neq \emptyset$ and $R(w_3) \cap \mathbf{A} \neq \emptyset$ but (2) $R_{|\mathbf{A}}(w_1) \cap R_{|\mathbf{A}}(w_2) = \emptyset$. Now, for contradiction, suppose that the Qualitative Thesis is valid on $\mathcal{F}$. Since that is so iff $\mathcal{F}$ obeys the Strict Union Constraint, we have that $(h(w_3) \cap \mathbf{A}) \subseteq R_{|\mathbf{A}}(w_1)$ and $(h(w_3) \cap \mathbf{A}) \subseteq R_{|\mathbf{A}}(w_2)$. But by supposition $R_{|\mathbf{A}}(w_1) \cap R_{|\mathbf{A}}(w_2) = \emptyset$. So $(h(w_3) \cap \mathbf{A}) = \emptyset$. But, since $R(w_3) \cap \mathbf{A} \neq \emptyset$, this contradicts Strict Non-Vacuity.

Fact 14. A minimal monotonic strict frame $\mathcal{F}$ validates Only Consistent Learning iff it validates No Opposite Materials.

Proof. Since neither Only Consistent Learning nor No Opposite Materials involve f or h, the proof is just as in Fact 9.

B The Qualitative Thesis in the Shifty Local Framework

Our language $\mathcal{L}$ is as before. A *shifty frame* $\mathcal{F}$ for $\mathcal{L}$ is a tuple $\langle W, R, R_{|\mathbf{A}}, f_\kappa \rangle$. f_κ is a shifty selection function, a function from $\mathcal{P}(W)$ to a selection function. The other elements of the tuple are as before.

A minimal shifty frame obeys S-Success, S-Non Vacuity, Inclusion and the following constraints on f_κ:

Success. $f_\kappa(w, \mathbf{A}) \subseteq \mathbf{A}$

Minimality. If $w \in \mathbf{A}$, then $w \in f_\kappa(w, \mathbf{A})$.

Non-Vacuity. If $\kappa \cap \mathbf{A} \neq \emptyset$ then $f_\kappa(w, \mathbf{A}) \neq \emptyset$.

A minimal monotonic shifty frame is a minimal shifty frame that also obeys Conditional Identity.

We recursively define truth at a world and a local context, i.e. a set of worlds in $\mathcal{W}$:

$[\![p]\!]^{\kappa,w}=1$ iff $w \in V(p)$

$[\![\neg\phi]\!]^{\kappa,w}=1$ iff $[\![\phi]\!]^{\kappa,w}=0$

$[\![\phi \wedge \psi]\!]^{\kappa,w}=1$ iff $[\![\phi]\!]^{\kappa,w}=[\![\psi]\!]^{\kappa,w}=1$

$[\![\phi > \psi]\!]^{\kappa,w}=1$ iff $f_\kappa(w,[\![\phi]\!]^\kappa) \subseteq [\![\psi]\!]^\kappa$

$[\![S\phi]\!]^{\kappa,w}=1$ iff $\forall w' \in R(w) : [\![\phi]\!]^{R(w),w'}=1$

$[\![S_\phi\psi]\!]^{\kappa,w}=1$ iff $\forall w' \in R_{[\![\phi]\!]^{R(w)}}(w) : [\![\psi]\!]^{R_{[\![\phi]\!]^{R(w)}},w'}=1$

where $[\![\phi]\!]^\kappa = \{w : [\![\phi]\!]^{\kappa,w}=1\}$.

Recall the following property of shifty frames from section 7:

Localized Indicative Constraint. If $\mathbf{A} \cap \kappa \neq \emptyset$, then $\forall w' \in \kappa : f_\kappa(w', \mathbf{A}) \subseteq \kappa$

We prove the following fact stated in the text:

Fact 15. If a minimal monotonic shifty frame $\mathcal{F}$ obeys the Local Indicative Constraint, then it validates QT.

Proof. Suppose the QT fails on a minimal monotonic shifty frame $\mathcal{F}$. Then for some κ and w, one of two cases obtains: i) $[\![\neg S\neg\phi]\!]^{\kappa,w}=1$, $[\![S(\phi > \psi)]\!]^{\kappa,w}=1$ and $[\![S(\phi \supset \psi)]\!]^{\kappa,w}=0$; or ii) $[\![\neg S\neg\phi]\!]^{\kappa,w}=1$, $[\![S(\phi \supset \psi)]\!]^{\kappa,w}=1$ and $[\![S(\phi > \psi)]\!]^{\kappa,w}=0$.

Case i) is ruled out by Minimality. For suppose i) obtains. Since $[\![S(\phi > \psi)]\!]^{\kappa,w}=1$, for all $w' \in R(w) : f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \subseteq [\![\psi]\!]^{R(w)}$. Since $[\![S(\phi \supset \psi)]\!]^{\kappa,w}=0$, there is some $w' \in R(w) : [\![\phi]\!]^{R(w),w'}=1$ and $[\![\psi]\!]^{R(w),w'}=0$. But by Minimality, this $w' \in f(w', [\![\phi]\!]^{R(w)})$. So $[\![\psi]\!]^{R(w),w'}=1$ after all. Contradiction.

In case ii), the Local Indicative Constraint fails. Since $[\![\neg S\neg\phi]\!]^{\kappa,w}=1$, there is some $w' \in R(w)$ s.t. $[\![\phi]\!]^{R(w),w'}=1$; so the antecedent of the Local Indicative Constraint is satisfied when $\kappa = R(w)$ and $\mathbf{A} = [\![\phi]\!]^{R(w)}$. Since $[\![S(\phi \supset \psi)]\!]^{\kappa,w}=1$, for all $w' \in R(w)$: either $[\![\phi]\!]^{R(w),w'}=0$ or $[\![\psi]\!]^{R(w),w'}=1$. Since $[\![S(\phi > \psi)]\!]^{\kappa,w}=0$, there is some $w' \in R(w)$ such that $f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \not\subseteq [\![\psi]\!]^{R(w)}$. Since by Success $f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \subseteq [\![\phi]\!]^{R(w)}$, it cannot be that $f_{R(w)}(w', [\![\phi]\!]^{R(w)}) \subseteq R(w)$. So the Indicative Constraint fails.

Note that the Localized Indicative Constraint is not *necessary* for validating QT: we only need the instances where $\kappa = R(w)$ for some w. But it seems to us that, from a semantic point of view, the more general principle is the more natural one.

References

[Adams, 1975] Adams, E. (1975). *The Logic of Conditionals*. D. Reidel, Dordrecht.

[Alchourrón et al., 1985] Alchourrón, C. E., Gärdenfors, P., and Makinson, D. (1985). On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic*, 50(2):510–530.

[Bacon, 2015] Bacon, A. (2015). Stalnaker's thesis in context. *Review of Symbolic Logic*, 8(1):131–163.

[Dekker, 1993] Dekker, P. (1993). *Transsentential meditations: Ups and Downs in Dynamic Semantics*. ILLC Dissertation Series, Amsterdam.

[Douven and Verbrugge, 2013] Douven, I. and Verbrugge, S. (2013). The probabilities of conditionals revisited. *Cognitive Science*, 37(4):711–730.

[Edgington, 1996] Edgington, D. (1996). Lowe on conditional probability. *Mind*, 105(420):617–630.

[Evans and Over, 2004] Evans, J. and Over, D. (2004). *If*. Oxford University Press, Oxford.

[Fitelson, 2015] Fitelson, B. (2015). The strongest possible lewisian triviality result. *Thought: A Journal of Philosophy*, 4(2):69–74.

[Gärdenfors, 1981] Gärdenfors, P. (1981). An epistemic approach to conditionals. *American Philosophical Quarterly*, 18(3):203–211.

[Gärdenfors, 1986] Gärdenfors, P. (1986). Belief revisions and the ramsey test for conditionals. *Philosophical Review*, 95(1):81–93.

[Gillies, 2004] Gillies, A. S. (2004). Epistemic conditionals and conditional epistemics. *Noûs*, 38(4):585–616.

[Gillies, 2009] Gillies, A. S. (2009). On truth-conditions for if (but not quite only if). *Philosophical Review*, 118(3):325–349.

[Grove, 1988] Grove, A. (1988). Two modellings for theory change. *Journal of Philosophical Logic*, 17(2):157–170.

[Hájek, 1989] Hájek, A. (1989). Probabilities of conditionals: Revisited.

[Hájek and Hall, 1994] Hájek, A. and Hall, N. (1994). The hypothesis of the conditional construal of conditional probability. In Eells, E., Skyrms, B., and Adams, E. W., editors, *Probability and Conditionals: Belief Revision and Rational Decision*, pages 75–113. Cambridge University Press, Cambridge.

[Harper, 1976] Harper, W. (1976). Ramsey test conditionals and iterated belief change. In Harper, W. and Hooker, C., editors, *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science, volume 1*, pages 302–306. Reidel, Dordrecht.

[Harper, 1975] Harper, W. L. (1975). Rational belief change, popper functions and counterfactuals. *Synthese*, 30(1-2):221–262.

[Hawthorne and Magidor, 2009] Hawthorne, J. and Magidor, O. (2009). Assertion, context, and epistemic accessibility. *Mind*, 118(470):377–397.

[Hawthorne and Magidor, 2010] Hawthorne, J. and Magidor, O. (2010). Assertion and epistemic opacity. *Mind*, 119(476):1087–1105.

[Heim, 1992] Heim, I. (1992). Presupposition projection and the semantics of attitude verbs. *Journal of semantics*, 9(3):183–221.

[Holguín, 2019] Holguín, B. (2019). Ramsey's thesis and introspection principles for knowledge and belief. Ms.

[Jeffrey and Edgington, 1991] Jeffrey, R. and Edgington, D. (1991). Matter-of-fact conditionals. *Aristotelian Society Supplementary Volume*, 65(1):161–209.

[Karttunen, 1974] Karttunen, L. (1974). Presupposition and linguistic context. *Theoretical linguistics*, 1(1-3):181–194.

[Khoo, 2019] Khoo, J. (2019). The meaning of *If*. Ms.

[Khoo and Mandelkern, 2019] Khoo, J. and Mandelkern, M. (2019). Triviality results and the relationship between logical and natural languages. *Mind*, 128(510):485–526.

[Kratzer, 2012] Kratzer, A. (2012). Conditionals. In *Modals and Conditionals*, chapter 4. Oxford University Press.

[Lewis, 1973] Lewis, D. (1973). *Counterfactuals*. Blackwell.

[Lewis, 1976] Lewis, D. (1976). Probabilities of conditionals and conditional probabilities. *Philosophical Review*, 85(3):297.

[Mandelkern, 2019a] Mandelkern, M. (2019a). Bounded modality. *Philosophical Review*, 128(1):1–61.

[Mandelkern, 2019b] Mandelkern, M. (2019b). If p, then p! Ms.

[Mandelkern and Khoo, ming] Mandelkern, M. and Khoo, J. (forthcoming). Against preservation. *Analysis*.

[McGee, 1989] McGee, V. (1989). Conditional probabilities and compounds of conditionals. *Philosophical Review*, 98(4):485–541.

[Pacuit, 2013] Pacuit, E. (2013). Dynamic epistemic logic i: Modeling knowledge and belief. *Philosophy Compass*, 8(9):798–814.

[Rothschild, 2013] Rothschild, D. (2013). Do indicative conditionals express propositions? *Noûs*, 47(1):49–68.

[Russell and Hawthorne, 2016] Russell, J. S. and Hawthorne, J. (2016). General dynamic triviality theorems. *Philosophical Review*, 125(3):307–339.

[Santorio, 2019] Santorio, P. (2019). Credence for indicative discourse. Ms.

[Schlenker, 2009] Schlenker, P. (2009). Local contexts. *Semantics and pragmatics*, 2:1–78.

[Schlenker, 2010] Schlenker, P. (2010). Presuppositions and local contexts. *Mind*, 119(474):377–391.

[Stalnaker, 1975a] Stalnaker, R. (1975a). Indicative conditionals. *Philosophia*, 5(3):269–286.

[Stalnaker, 1975b] Stalnaker, R. (1975b). Presuppositions. In *Contemporary Research in Philosophical Logic and Linguistic Semantics*, pages 31–41. Springer, dordrecht.

[Stalnaker, 1976] Stalnaker, R. (1976). Letter to $v$an $f$raassen. In Harper, W. and Hooker, C., editors, *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science, volume 1*, pages 302–306. Reidel, Dordrecht.

[Stalnaker, 1968] Stalnaker, R. C. (1968). A theory of conditionals. *American Philosophical Quarterly*, pages 98–112.

[Stalnaker, 1970] Stalnaker, R. C. (1970). Probability and conditionals. *Philosophy of Science*, 37(1):64–80.

[van Fraassen, 1976] van Fraassen, B. (1976). Probabilities of conditionals. In Harper, W. and Hooker, C., editors, *Foundations of probability theory, statistical inference, and statistical theories of science*. D. Reidel, Dordrecht.

[von Fintel, 1998] von Fintel, K. (1998). The presupposition of subjunctive conditionals. In Sauerland, U. and Percus, O., editors, *MIT Working Papers in Linguistics 25*, pages 29–44. MITWP, Cambridge, MA.

[Willer, 2017] Willer, M. (2017). Lessons from sobel sequences. *Semantics and Pragmatics*, 10.

[Williamson, 2000] Williamson, T. (2000). *Knowledge and its Limits*. Oxford University Press, Oxford.

[Yalcin, 2007] Yalcin, S. (2007). Epistemic modals. *Mind*, 116.